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Generating non-classical states from spin coherent states via interaction with ancillary spins

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ABSTRACT

The generation of non-classical states of large quantum systems has attracted much interest from a foundational perspective, but also because of the significant potential of such states in emerging quantum technologies. In this paper we consider the possibility of generating non-classical states of a system of spins by interaction with an ancillary system, starting from an easily prepared initial state. We extend previous results for an ancillary system comprising a single spin to bigger ancillary systems and the interaction strength is enhanced by a factor of the number of ancillary spins. Depending on initial conditions, we find – by a combination of approximation and numerics – that the system of spins can evolve to spin cat states, spin squeezed states or to multiple cat states. We also discuss some candidate systems for implementation of the Hamiltonian necessary to generate these non-classical states.

1. Introduction

The apparent conflict between the classical world of our everyday experience and the underlying quantum reality has been widely discussed since the beginnings of quantum physics. A prominent line of research is the effort to create entanglement between a macroscopic quantum system and a microscopic system along the lines of the original Schrödinger cat thought experiment [1,2]. The generation of macroscopic quantum states is also interesting from a technological point of view. In optical systems, for example, it is known that various macroscopic non-classical states can be exploited to give a significant improvement in the precision of phase estimation [3,4]. Along these lines, many proof-of-principle experiments have been demonstrated with optical systems [5]. However, hybrid quantum systems might be eventually needed to extract quantum advantages in practical quantum technology because alternative physical set-ups could provide different advantages.

Recently, continuous-variable (CV) superposed/entangled states have shown their potential in various optical and photonic experiments [6] and the use of CV entangled states can be robust in practical quantum metrology [7–9]. Recently, "micro-macro" entangled states have been implemented from a path-entangled single photon state [10,11]. The beam splitting interaction, by putting a vacuum in one mode and a single photon Fock state in the other, followed by a displacement in one of the modes leads to the micro-macro entangled state $D_B(\alpha)(|1\rangle_A|0\rangle_B+|0\rangle_A|1\rangle_B)/\sqrt{2}$ where $D_B(\alpha)$ is a displacement operator with amplitude α in mode B [11].

Here, we consider the generation of non-classical states of two interacting systems A and B where system A is a collection of N_A spin-1/2 particles and ancillary system B is a collection of N_B spins. Spin states have previously been considered as a way of storing a qubit (i.e., two orthogonal collective spin states are used as computational basis states of an effective qubit [12–15]) but can be naturally utilised for CV quantum information processing by creating CV entangled states in a spin system. We assume that the initial state consists of each of the N_A qubits in the same pure state, an easily prepared state in principle. In [16,17] it was shown that various CV states (e.g., spin cat states, multiple cat states, and spin squeezed states) can be generated from a spin coherent state (SCS) for $N_B=1$. By a combination of approximation and numerics, we investigate cases when $N_B>1$. An advantage of $N_B>1$ compared to $N_B=1$ is faster preparation times of the non-classical CV states.

This paper is organised as follows. In Section 2, we give the interaction Hamiltonian and show that it has several well-known Hamiltonians as limits. In Section 3 we present short-time approximations for the dynamics of the model for two different initial states of spin system A. In the first case, we show that for a carefully chosen initial state of the ancillary system B, the system A evolves to a superposition of two spin coherent states, a spin "Schrodinger cat" state. In the second approximation we show that spin system A evolves to spin squeezed states. We also give numerical evidence that at later times, beyond the restriction of

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the approximation, the combined system AB can evolve to a superposition of many spin coherent states ("multiple cat states") of the combined system. We suggest an ansatz Hamiltonian that predicts the gross features of the dynamics in this case.

In Section 4, we discuss a beam-splitter (BS) type interaction between two distinct systems of spins. If a non-classical SCS interacts with a typical SCS, the resultant state can be understood as an entangled CV SCS in two modes. This has potential to be implemented in Bose–Einstein condensates (BECs) and Nitrogen-vacancy centres (NV-centres) with superconducting systems. Finally, we summarise the results in Section 5.

2. Spin Hamiltonian model

Let us assume that system A is a collection of N_A spins and system B is that of N_B spins. We consider a Hamiltonian of the form

$$\hat{H}(N_A, N_B) = \omega_A\left(\hat{J}_A^z + \frac{N_A}{2}\right) + \omega_B\left(\hat{J}_B^z + \frac{N_B}{2}\right) + \lambda(\hat{J}_A^+\hat{J}_B^- + \hat{J}_A^-\hat{J}_B^+), \tag{1}$$

where the J-operators on system A are defined as

$$\hat{J}_A^\mu = \frac{1}{2}\sum_{i=0}^{N_A}\hat{\sigma}_{(i)}^\mu; \quad \hat{J}_A^\pm = \sum_{i=0}^{N_A}\hat{\sigma}_{(i)}^\pm; \quad \hat{J}_A^2 = \sum_\mu(\hat{J}_A^\mu)^2, \tag{2}$$

where $\hat{\sigma}^\mu$ are the Pauli operators for the individual spins of A with $\mu \in \{x, y, z\}$. The J-operators for B are defined in the same way.

The *Dicke states* are the set of simultaneous eigenstates of the commuting operators $\hat{J}_A^2$ and $\hat{J}_A^z$, and are denoted by $|j, n-j\rangle_A$ where

$$\hat{J}_A^2|j, n-j\rangle_A = j(j+1)|j, n-j\rangle_A; \quad \hat{J}_A^z|j, n-j\rangle_A = (n-j)|j, n-j\rangle_A \tag{3}$$

for $j \in \{0, 1, \dots, N_A/2\}$ if N_A is even, $j \in \{\frac{1}{2}, \frac{3}{2}, \dots, N_A/2\}$ if N_A is odd, and $n \in \{0, 1, \dots, 2j\}$. States in the $j = N_A/2$ eigenspace of the N_A spin system are totally symmetric with respect to exchange of any two spins. In particular, the $j = N_A/2$ Dicke states are totally symmetric:

$$\left|\frac{N_A}{2}, n - \frac{N_A}{2}\right\rangle_A = \binom{N_A}{n}^{-1/2}\sum_{\text{permutations}}|\downarrow^{\otimes(N_A-n)}\uparrow^{\otimes n}\rangle, \tag{4}$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are eigenstates of σ^z for a single spin. The N_A+1 Dicke states ($n \in \{0, 1, \dots, N_A\}$) are a basis for the $j = N_A/2$ eigenspace (this is true only for this eigenspace, the one associated with the maximal value of j). In what follows we restrict to the $j = N_A/2$ eigenspace and the Dicke state in Eq. (4) is written as $|N_A/2, n - N_A/2\rangle_A \equiv |n\rangle_A$ for simplicity.

The $\hat{J}_A^\pm$ operators have the effect of raising and lowering the n index of Dicke states:

$$\hat{J}_A^+|n\rangle_A = \sqrt{(n+1)(N_A-n)}|n+1\rangle_A, \tag{5}$$

$$\hat{J}_A^-|n\rangle_A = \sqrt{n(N_A-n+1)}|n-1\rangle_A. \tag{6}$$

These raising and lowering operators have a superficial similarity to the creation and annihilation operators of a bosonic field mode,

$$\hat{a}^\dagger|\overline{n}\rangle = \sqrt{n+1}\,|\overline{n+1}\rangle; \quad \hat{a}|\overline{n}\rangle = \sqrt{n}\,|\overline{n-1}\rangle, \tag{7}$$

where $|\overline{n}\rangle$ are Fock states, eigenstates of $\hat{a}^\dagger\hat{a}$ (the bar above n indicates a state of the field mode rather than a state of the finite spin system). In fact, it is not difficult to see that if we identify the Dicke state $\lim_{N_A\to\infty}|n\rangle_A$ with the Fock state $|\overline{n}\rangle$ of a bosonic mode, then

$$\lim_{N_A\to\infty}\frac{\hat{J}_A^+}{\sqrt{N_A}} = \hat{a}^\dagger; \quad \lim_{N_A\to\infty}\frac{\hat{J}_A^-}{\sqrt{N_A}} = \hat{a}, \tag{8}$$

and $\hat{J}_A^\pm/\sqrt{N_A}$ obey the bosonic commutation relations:

$$\lim_{N_A\to\infty}\left[\frac{\hat{J}_A^-}{\sqrt{N_A}}, \frac{\hat{J}_A^+}{\sqrt{N_A}}\right] = 1. \tag{9}$$

This is the *bosonic limit* of the spin raising and lowering operators. If N_A is finite then we have

$$\left[\frac{\hat{J}_A^-}{\sqrt{N_A}}, \frac{\hat{J}_A^+}{\sqrt{N_A}}\right] = 1 - \frac{2}{N_A}\left(\hat{J}_A^z + \frac{N_A}{2}\right), \tag{10}$$

and the spin raising and lowering operators approximately satisfy the bosonic commutation relations only if the second term on the right-hand side of Eq. (10) can be neglected. For N_A finite we also have the Holstein–Primikoff transformations [18] that relate the J-operators to the bosonic operators:

$$\frac{\hat{J}_A^-}{\sqrt{N_A}} = \sqrt{1 - \frac{\hat{a}^\dagger\hat{a}}{N_A}}\,\hat{a}; \quad \frac{\hat{J}_A^+}{\sqrt{N_A}} = \hat{a}^\dagger\sqrt{1 - \frac{\hat{a}^\dagger\hat{a}}{N_A}}; \quad \hat{J}_A^z = \hat{a}^\dagger\hat{a} - \frac{N_A}{2}. \tag{11}$$

As in Eq. (10), if the $\hat{a}^\dagger\hat{a}/N_A$ contributions under the square roots in Eq. (11) can be neglected, the N_A spin system (in the $j = N_A/2$ subspace) is well approximated as a bosonic mode.

The model Hamiltonian (1) has a number of other interesting models as special limits. To see this it is first useful to renormalise the interaction parameter to $\lambda = \tilde{\lambda}/\sqrt{N_A N_B}$ so that we get sensible results after taking limits. Then, for example, if we take the $N_A \to \infty$ limit and choose $N_B = 1$ we are left with the familiar Jaynes–Cummings Hamiltonian for the interaction of a bosonic mode with a two level system:

$$\hat{H}(\infty, 1) = \omega_A\hat{a}_A^\dagger\hat{a}_A + \frac{\omega_B}{2}(\hat{\sigma}_B^z + 1) + \tilde{\lambda}(\hat{a}_A\hat{\sigma}_B^+ + \hat{a}_A^\dagger\hat{\sigma}_B^-). \tag{12}$$

If we let $N_A \to \infty$ and allow N_B to be some finite number we have the Tavis–Cummings Hamiltonian:

$$\hat{H}(\infty, N_B) = \omega_A\hat{a}_A^\dagger\hat{a}_A + \omega_B\left(\hat{J}_B^z + \frac{N_B}{2}\right) + \frac{\tilde{\lambda}}{\sqrt{N_B}}(\hat{a}_A\hat{J}_B^+ + \hat{a}_A^\dagger\hat{J}_B^-). \tag{13}$$

If we take both $N_A \to \infty$ and $N_B \to \infty$ we get

$$\hat{H}(\infty, \infty) = \omega_A\hat{a}_A^\dagger\hat{a}_A + \omega_B\hat{b}^\dagger\hat{b}_B + \tilde{\lambda}(\hat{a}_A\hat{b}_B^\dagger + \hat{a}_A^\dagger\hat{b}_B), \tag{14}$$

the Hamiltonian for an exchange interaction between two bosonic modes.

Each of these interaction Hamiltonians can be used – in principle – to generate macroscopic superposition states of, say, system A, and/or macroscopic entangled states of AB, starting from easily prepared initial states of A. The on-resonance Jaynes–Cummings model, for instance, with an initial mesoscopic coherent state and an appropriately chosen initial qubit state, evolves to a Schrödinger cat state of the field mode at a quarter of the revival time (via an entangled state of the field and the atom) [19]. The on-resonance Tavis–Cummings Hamiltonian can be applied to generate the same Schrödinger cat state with a shorter evolution time, but with the cost that the N_B qubits must be initially in a GHZ-type state [20].

Transforming the Hamiltonian (1) to the interaction picture with respect to the free Hamiltonian $\hat{H}_0 = \omega_B(\hat{J}_A^z + \hat{J}_B^z + (N_A + N_B)/2)$ gives the interaction picture Hamiltonian

$$\hat{H}_I(N_A, N_B) = \Delta\left(\hat{J}_A^z + \frac{N_A}{2}\right) + \lambda(\hat{J}_A^+\hat{J}_B^- + \hat{J}_A^-\hat{J}_B^+), \tag{15}$$

where $\Delta = \omega_A - \omega_B$ is the detuning. On resonance ($\omega_A = \omega_B$) this reduces to

$$\hat{H}_I(N_A, N_B) = \lambda(\hat{J}_A^+\hat{J}_B^- + \hat{J}_A^-\hat{J}_B^+). \tag{16}$$

This interaction term allows for a collective, coherent excitation to be exchanged between system A and system B.