Optics Communications 337 (2015) 79–82

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Coherent-state optical qudit cluster state generation and teleportation via homodyne detection

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ARTICLE INFO

Article history:
Received 16 May 2014
Received in revised form
24 June 2014
Accepted 4 July 2014
Available online 18 July 2014

Keywords:
Qudit
Cluster state
Cross-Kerr
Teleportation

ABSTRACT

Defining a computational basis of *pseudo-number* states, we interpret a coherent state of large amplitude, $|\alpha| \gg d/2\pi$, as a qudit — a d-level quantum system — that is an *even* (meaning same size of amplitudes) superposition of d *pseudo-number* basis states. A pair of such coherent-state qudits can be maximally entangled by *generalized* Controlled-Z operation that is based on cross-Kerr nonlinearity, which can be weak for large d. Hence, a coherent-state optical qudit cluster state can be prepared by repetitive application of the *generalized* Controlled-Z operation to a set of coherent states. We thus propose an optical qudit teleportation as a simple demonstration of cluster state quantum computation.

1. Introduction

Quantum computation is expected to speed up some computational problems exponentially and some others quadratically compared to the best known digital computation [1]. Even though many experimental proposals of quantum computers are made, there seem to be many obstacles such as decoherence, scalability, inaccurate operation, and so on [2]. There are two approaches to the quantum computing — one that *molds* quantum state while the other *sculptures* it. Molding of quantum states lies in the heart of original schemes for quantum computers that are based on quantum circuits. In these schemes, one prepares an initial quantum state made of many qubits and applies quantum operations on it, which are followed by a measurement that leads to the result.

Raussendorf and Briegel proposed a special quantum entanglement called a cluster state [3] and went on proposing cluster state quantum computation with Browne [4]: you prepare a cluster state, a giant maximally entangled state of many qubits, and just measure each qubit away feedforwardly which means measurements are done based on previous measurement results — effectively *sculpturing* the state. To make a quantum cluster state, prepare qubits as $|+\rangle$, even (meaning same size of amplitudes) superposition of computational basis kets $|0\rangle$ and $|1\rangle$ at each lattice point and apply $\mathcal{CZ}_{ct}$ (c is an index for the control qubit and t is for the target qubit) operations on all neighboring qubits in the lattice.

Even though the number of required qubits is polynomially larger than quantum circuit model, cluster state quantum computation is simpler since only single qubit measurements are needed once a cluster state is prepared.

Based on Knill, Laflamme, and Milburn's all-optical quantum computing [5] and Raussendorf, Browne, and Briegel's cluster state quantum computing [4], Nielsen and Dawson proposed optical cluster state quantum computing [6,7]. One important demerit of the proposal might be the probabilistic nature of linear optical gating. Nonlinear optics can be used to generate quantum optical entanglements [8] and generation of optical qubit cluster states are proposed [9–11]. These schemes, however, need impractically large nonlinearities. Instead of qubits with two basis states, quantum computation using cluster states of d-state quantum systems or qudits has been proposed with the possibility of realization with high-dimensional Ising model [12].

Many proposals of using coherent states and/or nonlinear optical interactions for quantum information processing have been made [13]. In this paper, optical coherent states are interpreted as qudits of *even* (meaning same size of amplitudes) superposition of basis states and these qudits are deterministically entangled into qudit cluster states using cross-Kerr nonlinear interaction. Qudit cluster states might be used for quantum computation or quantum communication network.

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http://dx.doi.org/10.1016/j.optcom.2014.07.016

2. Optical coherent states as qudits

Here we propose a simple deterministic optical scheme to generate a cluster state of qudits. First we notice that the infinite Taylor series of an exponential function can be decomposed into d infinite partial sums each of which asymptotically approaches e^x/d for any finite integer d as can be seen in the following:

$$e^x=\sum_{k=0}^{d-1} f_k(x) \quad \text{with } f_k(x)=\sum_{m=0}^{\infty}\frac{x^{k+md}}{(k+md)!} \tag{1}$$

where

$$\lim_{x\to\infty}\frac{f_k(x)}{e^x}=\frac{1}{d} \quad \text{for } k=0,\ldots,d-1. \tag{2}$$

2.1. Qudits in pseudo-number basis and pseudo-phase basis

In a similar manner a coherent state $|\alpha\rangle$ can be interpreted as a qudit that is evenly (meaning same size of amplitudes) superposed in a computational basis with d basis ket vectors when $|\alpha|\gg d/2\pi$ and this condition is assumed throughout this paper:

$$|\alpha\rangle=e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle=\frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|k_d\rangle \tag{3}$$

with orthonormalized computational basis kets

$$|k_d\rangle=\sqrt{d}e^{-|\alpha|^2/2}\sum_{m=0}^{\infty}\frac{\alpha^{k+md}|k+md\rangle}{\sqrt{(k+md)!}} \quad \text{for } k=0,\ldots,d-1 \tag{4}$$

that we call *pseudo-number* states since each ket is made of photon number states with definite modulo-d number of photons.

By applying a *generalized* Hadamard transformation $\hat{H}$

$$\hat{H}=\frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}\sum_{l=0}^{d-1}\omega^{kl}|k_d\rangle\langle l_d| \tag{5}$$

on computational basis ket $|k_d\rangle$'s, we can get conjugated basis kets

$$|\widetilde{l_d}\rangle=\hat{H}|l_d\rangle=\frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}\omega^{lk}|k_d\rangle \quad \text{for } l=0,\ldots,d-1 \tag{6}$$

with $\omega=e^{2\pi i/d}$.

These conjugated basis kets are nothing but the coherent states

$$|\widetilde{l_d}\rangle=|\omega^l\alpha\rangle$$

that we call *pseudo-phase* states since each ket of this basis is a coherent state centered at a definite optical phase.

A *generalized* $\hat{Z}$ operator for qudits is defined as

$$\hat{Z}=\sum_{k=0}^{d-1}\omega^k|k_d\rangle\langle k_d|=\omega^{\hat{n}}$$

with a photon number operator $\hat{n}$ and a *generalized* Controlled-Z operator, $\hat{Z}_{\rm ct}$, is defined as

$$\hat{Z}_{\rm ct}=\sum_{l=0}^{d-1}|l_d\rangle_{\rm c\,c}\langle l_d|\otimes\hat{Z}_{\rm t}^{l}=\omega^{\hat{n}_{\rm c}\hat{n}_{\rm t}}$$

with c and t for control and target qudits respectively.

2.2. Qudit cluster state generation through optical operations

A *generalized* $\hat{Z}$ operator can be easily implemented by a phase shifter $e^{(2\pi i/d)\hat{n}}$ with photon number operator $\hat{n}$, and a *generalized* Controlled-Z operator, $\hat{Z}_{\rm ct}$, can be realized by cross-Kerr medium. If the cross-Kerr interaction with Hamiltonian $H=-\hbar\chi\hat{n}_1\hat{n}_2$ is applied to two-coherent-state input $|\alpha\rangle_1|\alpha\rangle_2$ for time $t=2\pi/d\chi$, we can get

$$\begin{aligned}e^{(2\pi i/d)\hat{n}_1\hat{n}_2}|\alpha\rangle_1|\alpha\rangle_2 &=\hat{Z}_{12}\left(\frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|k_d\rangle\right)_1\left(\frac{1}{\sqrt{d}}\sum_{l=0}^{d-1}|l_d\rangle\right)_2\\ &=\frac{1}{d}\sum_{k=0}^{d-1}\sum_{l=0}^{d-1}\omega^{kl}|k_d\rangle_1|l_d\rangle_2\\ &=\frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|k_d\rangle_1|\widetilde{k_d}\rangle_2 \quad \text{or} \quad \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|\widetilde{k_d}\rangle_1|k_d\rangle_2\end{aligned} \tag{7}$$

which is a maximally entangled state of two qudits, that is, we can generate a maximal entanglement of *pseudo-phase* and *pseudo-number* states by simply applying cross-Kerr interaction on two coherent beams. The larger the d, the easier the implementation of $\hat{Z}_{\rm ct}$ of qudits is since it can be achieved with smaller $\chi t=2\pi/d$. Van Enk [14] proposed an idea of making entangled coherent states using self-Kerr interaction. Van Enk's entangled state is the type of *pseudo-number/pseudo-number* or *pseudo-phase/pseudo-phase*, i.e.,

$$\frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|k_d\rangle|k_d\rangle=\frac{1}{\sqrt{d}}\sum_{l=0}^{d-1}|\widetilde{l_d}\rangle|\widetilde{l_d}\rangle.$$

Cheong and Lee proposed the use of cross-Kerr interaction for making d-dimensional entangled coherent states [15] as in this paper. Van Enk's proposal cannot be extended beyond two qudit entanglement using only self-Kerr interaction and Cheong and Lee's proposal was not extended beyond two qudit entanglement.

If we apply $\hat{Z}_{\rm ct}$ to all neighboring coherent states as illustrated in Fig. 1, we can get a cluster state of qudits

$$\prod_{\langle \rm p,q\rangle}\omega^{\hat{n}_{\rm p}\hat{n}_{\rm q}}\prod_{\rm r\in lattice}|\alpha\rangle_{\rm r}$$

where $\langle \rm p,q\rangle$ represents neighbors in the lattice. Since all the Controlled-Z's are commuting with each other, the order of the operations is not important.

It used to be believed that two-qubit operations are the most difficult part and single qubit operations are relatively easier in quantum information processing. Now contrary to this conventional wisdom of qubit processing, Controlled-Z of two qudits and preparation of cluster states of optical qudits gets easier as the dimension d gets larger. A *generalized* $\hat{X}$ operator can be defined as

$$\hat{X}=\sum_{k=0}^{d-1}|(k-1)_d\rangle\langle k_d| \quad \text{with } |-1_d\rangle=|(d-1)_d\rangle,$$

which is similar to Pegg–Barnett *phase* operator [16] and could be called *pseudo-phase* operator. In *pseudo-phase* basis it can be written as

$$\hat{X}=\sum_{l=0}^{d-1}\omega^l|\widetilde{l_d}\rangle\langle\widetilde{l_d}|$$

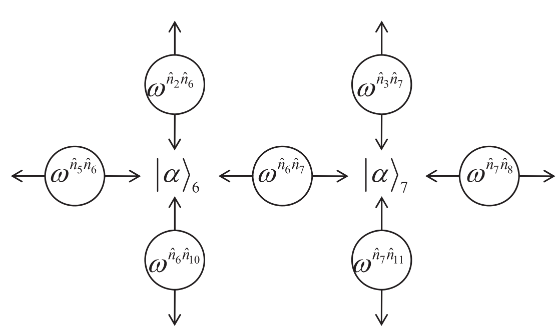

Fig. 1. Generating a cluster state of coherent-state optical qudits. $\omega^{\hat{n}_1\hat{n}_2}=e^{(2\pi i/d)\hat{n}_1\hat{n}_2}=\hat{Z}_{12}$ and so on.