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Assessments of macroscopicity for quantum optical states

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ABSTRACT

With the slow but constant progress in the coherent control of quantum systems, it is now possible to create large quantum superpositions. There has therefore been an increased interest in quantifying any claims of macroscopicity. We attempt here to motivate three criteria which we believe should enter in the assessment of macroscopic quantumness: The number of quantum fluctuation photons, the purity of the states, and the ease with which the branches making up the state can be distinguished.

1. Introduction

More than eighty years after its inception, quantum mechanics has become firmly established as a reliable model of the physical world. Even counter intuitive notions such as Schrödinger's cat and wave-particle duality have trickled into the layman's vocabulary. Yet, to this day and even within the physics community, the coherent superposition of macroscopic objects still seems to intrigue more than that of microscopic ones. One obvious reason for this is that, because of decoherence, large numbers of particles are difficult to shepherd into coherent ensembles. However, decoherence on its own does not account for the vagueness surrounding the macroscopicity buzzword [1].

Several experiments, especially in solid state and atomic setups at cryogenic regimes, have exhibited quantized or coherent behavior of macroscopic scales [2–4]. In quantum optics, coherent state superpositions, the so-called Schrödinger cat states of light, have been generated and thoroughly studied for nearly a decade [5–8]. In view of these advances, the question of macroscopicity has shifted to a quantitative one: What observables make up the "size" of a quantum state? Several equally valid measures for this were proposed over the years [9–19]. Our purpose here is to give a bigger picture of the various prerequisites that macroscopicity entails. Indeed, the word "macroscopicity" has a dual etymology with "macro" meaning large, and "scope" alluding to an observer-dependent perspective. If, in addition, we talk of the *quantum* macroscopicity of a system, we also expect that it exhibits quantum coherence. The criterion for quantum macroscopicity is thus three-fold: One should assert that a system is (1) large, (2) quantum, and (3) demonstrably composed of macroscopically distinct branches in at least some of its subsystems.

The outline of this paper is as follows. We begin by treating points (1) and (2) above in Section 2, where we present a measure for the size of pure states which consists of the number $\mathcal{N}$ of fluctuation photons. Such a measure is objective in the sense that it is independent of the measurement process. We also give a brief reminder that the quantumness of a state is related to its purity and that the inclusion of purity in the macroscopicity measure is necessary, albeit non-trivial. Section 3 discusses the observer's ability to distinguish mixed states. This is formalized with a distinguishability factor $\mathcal{D}$ which is then combined with $\mathcal{N}$ to produce what we shall refer to as the subjective—perceived—macroscopicity $\mathcal{M}$. By the same token, we emphasize that distinguishability is fundamentally ill-defined for the branches of a coherent superposition.

2. Objective macroscopicity

Our heuristic approach to macroscopicity begins with the phase space representation of physical states. Consider a classical state tracing a trajectory in phase space under some potential. It is represented by a geometrical point whose distance from the origin reflects how excited it is. In quantum mechanics, this point acquires a continuous pseudo-probability distribution—typically a Gaussian of finite width—of which it becomes the centroid. The canonical coordinates of the centroid are the same as in the classical picture [20]; the first moment of the distribution is therefore unlikely to describe quantum properties. The second moment, on the other hand, arises from a coherent set of quantum

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fluctuations. It is these fluctuations, which amount to the quantum noise of the distribution, that are of interest. This means that, in effect, a coherent state of light is as macroscopic as the vacuum since the two only differ by their centroid's position. Even if the coherent state contains a larger number of photons, the effective number of them that contributes to quantum fluctuations is the same as that of the vacuum, namely zero. All the other photons can be considered as nothing more than a classical offset with no coherence content.

From the motivation outlined above, we therefore propound that the macroscopicity of a quantum optical state is quantified by the mean number of photons minimized over all possible displacement operations [29]. In other words, we define the macroscopicity as the number of photons associated with the fluctuations within a pure state $|\psi\rangle$:

$$\begin{aligned}\mathcal{N}(|\psi\rangle) &= \langle \hat{n}_{\text{fluct.}} \rangle_{|\psi\rangle} \\ &= \langle \hat{n} \rangle - \langle \hat{n}_{\text{centroid}} \rangle \\ &= \frac{1}{2}(\text{var}(x) + \text{var}(p) - 1), \end{aligned} \tag{1}$$

with $\hat{n} = \frac{1}{2}(\hat{x}^2 + \hat{p}^2 - 1)$ and $\langle \hat{n}_{\text{centroid}} \rangle = \langle \hat{a} \rangle \langle \hat{a}^\dagger \rangle = \frac{1}{2}(\langle \hat{x} \rangle^2 + \langle \hat{p} \rangle^2)$. The measure $\mathcal{N}$ is objective in the sense that it is expressed in physical units of fluctuation photons with no dependence on the measurement process. Further below, we shall also present a subjective—observer-dependent—version of it.

It is worth mentioning that, for pure states, the macroscopicity $\mathcal{N}$ coincides with that of Lee and Jeong [12], who arrived at their own measure from an entirely different motivation and which in turn matched some earlier results by Dür et al. [11]. We take this convergence of results as a strong indication that our heuristic described above is valid.

A warning is in order at this point regarding a critical caveat: the distributions of which we compute the second moment should be made up exclusively of coherent excitations. In other words, the derivation leading up to (1) only reveals genuine *quantum fluctuation* photons provided the state under consideration has unit purity. Failing this, we lose track of whether the variance in the canonical coordinates is of quantum or classical origin since both distributions are blended indiscriminately into one and the same Wigner function. This is illustrated with the example of coherent state superpositions and mixtures in Fig. 1. The generalization of (1) which discerns the quantum second-moments from the classical ones is a non-trivial matter which we shall not attempt to tackle here. For the sake of simplicity, we shall therefore limit our discussion to pure states. (For a treatment of mixed states, we refer to the work of Lee and Jeong [12], who provide a general and intuitive strategy.)

3. Subjective macroscopicity

We have so far presented the quantum size of an optical system as an objective measure that is observer independent. However, in the quantum optics community, the notion of macroscopicity is often associated with the subjective ability of an observer to distinguish the branches with a coarse-grained detector [13–15]. Using a “classical” detector such as the naked eye or a coarse grained intensity detector one should be able to infer any one of the branches of the quantum state. The underpinning idea is that coarse-graining, for being insensitive to microscopic observables, can only discern macroscopically separated eigenvalues.

It is therefore useful to define a subjective macroscopicity measure that involves the ability to distinguish pre-specified branches of a macroscopic quantum state. This contains not only information about how large a state is (the objective macroscopicity) but also information about how far apart its branches are from one another from the perspective of the observer. This follows the original spirit of Schrödinger's thought experiment.

3.1. The notion of distinguishability

We shall first elaborate on the notion of distinguishability in order to later incorporate it in a subjective measure of macroscopicity. The two concepts are often interlinked in the literature, as exemplified by the work of Korsbakken et al. in [13]. A related strategy was followed by Sekatski et al. [14,15] who define macroscopicity by the ability to discriminate the branches using a classical-like intensity detector which cannot resolve photon numbers. Such a discrimination task of macroscopically distinct branches by using ideal homodyne detectors was considered in Ref. [21]. Strongly motivated by these works (in particular by the work of Sekatski et al.), we shall consider in this section the notion of distinguishability using a noisy detector and present various relevant examples.

Recall Schrödinger's original thought experiment: A macroscopic cat, which is entangled with an atomic qubit $\{|\uparrow\rangle, |\downarrow\rangle\}$, is collapsed upon observation into either one of the two orthogonal states $|\text{DEAD}\rangle$ or $|\text{ALIVE}\rangle$. For this simple two-level, two-mode system, the notion of distinguishability is essentially a measure of our confidence in being able to identify in a single-shot that the cat is either dead or alive. We shall express this confidence level as

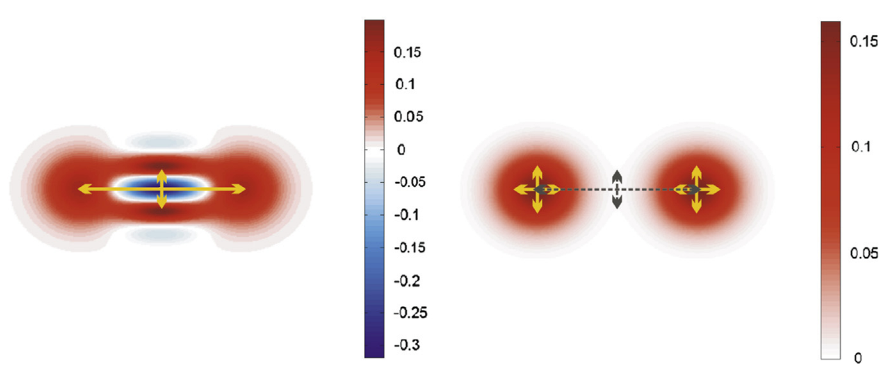

Fig. 1. Wigner profiles of a coherent state superposition $|\alpha\rangle - |-\alpha\rangle$ (left) and a coherent state mixture $|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$ (right) for $\alpha = 1.5$. The x- and p-variances are represented schematically by the orthogonal double arrows. The crucial difference between the superposition and the mixture is that the *coherent* variance—the one that arises from quantum fluctuations, not classical statistics—is much smaller for the mixtures. Whereas it spans both lobes of the Wigner function for the superposition, its extent in the mixture is merely that of either coherent state $|\pm\alpha\rangle$. This “genuine quantum” variance is represented by the yellow arrows and that is the one that should enter in the macroscopicity. Since both quantum and classical statistics get blended together in the Wigner function, a blind application of Eq. (1) will yield the wrong result for mixed states as it will mistake the overall variance (dotted gray) for a quantum variance. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)