ARTICLE IN PRESS

Optics Communications ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Macroscopic quantum information processing using spin coherent states

Tim Byrnes [a,*], Daniel Rosseau [a], Megha Khosla [b,c], Alexey Pyrkov [d], Andreas Thomasen [a], Tetsuya Mukai [e], Shinsuke Koyama [f], Ahmed Abdelrahman [g], Ebubechukwu Ilo-Okeke [a,h]

[a] National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan
[b] Max Planck Institute for Informatics, Saarbrücken, Germany
[c] Saarland University, Saarbrücken, Germany
[d] Institute of Problems of Chemical Physics RAS, Acad. Semenov av., 1, Chernogolovka 142432, Russia
[e] NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, Kanagawa 243-0198, Japan
[f] The Institute of Statistical Mathematics, 10-3 Midori-cho, Tachikawa, Tokyo 190-8562, Japan
[g] Department of Physics, University of California, Berkeley, CA 94720, USA
[h] Department of Physics, School of Science, Federal University of Technology, P.M.B. 1526, Owerri, Imo State 460001, Nigeria

ARTICLE INFO

Article history:
Received 5 February 2014
Received in revised form
7 August 2014
Accepted 8 August 2014

Keywords:
Bose–Einstein condensates
Quantum information
Spin coherent states
Quantum computing

ABSTRACT

Previously a new scheme of quantum information processing based on spin coherent states of two component Bose–Einstein condensates was proposed (Byrnes et al. Phys. Rev. A 85, 40306(R)). In this paper we give a more detailed exposition of the scheme, expanding on several aspects that were not discussed in full previously. The basic concept of the scheme is that spin coherent states are used instead of qubits to encode qubit information, and manipulated using collective spin operators. The scheme goes beyond the continuous variable regime such that the full space of the Bloch sphere is used. We construct a general framework for quantum algorithms to be executed using multiple spin coherent states, which are individually controlled. We illustrate the scheme by applications to quantum information protocols, and discuss possible experimental implementations. Decoherence effects are analyzed under both general conditions and for the experimental implementation proposed.

1. Introduction

Bose–Einstein condensation was first achieved in 1995 for ultracold atoms [1,2], as well as a variety of different physical systems, ranging from exciton-polaritons [4], magnons [5], photons [6], and superfluid Helium [7]. For atomic Bose–Einstein condensates (BECs), atom chip technology has made possible the miniaturization of traps on the micrometer scale, allowing for the possibility of the individual formation and control of many BECs [8]. Due to the long coherence times of cold atoms, a natural application for such systems is quantum information processing, ranging from such tasks as quantum metrology [3], quantum simulation [9], and quantum computing.

Recently, two component BECs were realized on atom chips realizing full control on the Bloch sphere and spin squeezing [11,10,12]. The primary application for such two component BECs is currently thought to be for quantum metrology and chip based clocks. Here we discuss its applications towards quantum computation. In particular we review a new approach to quantum information processing based on spin coherent states of two component BECs, originally proposed in Ref. [14]. While BECs have been considered for quantum computation in the past in works such as Ref. [13], the results have shown to be generally unfavorable for these purposes due to enhanced decoherence effects due to the large number of bosons N in the BEC. The basic idea of the scheme in Ref. [14] is to take advantage of the analogous state structure of spin coherent states on the Bloch sphere as qubits. The state of a qubit at a particular location on the Bloch sphere is encoded as a spin coherent state with the same parameters on the Bloch sphere. Manipulations of the state then proceed by applying collective spin operators $S^{x,y,z}$ and the entangling operations S^zS^z. Using this particular encoding of the quantum information, largely mitigates the problem of decoherence as found in Ref. [13]. We develop the framework for quantum computation using this encoding, illustrated with several quantum algorithms. We also analyze the effects of decoherence from several standpoints and discuss the scheme's performance under a variety of conditions.

* Corresponding author. +81 3 4212 2000.
E-mail address: tbyrnes@nii.ac.jp (T. Byrnes).
URL: http://nii.timbyrnes.net (T. Byrnes).

http://dx.doi.org/10.1016/j.optcom.2014.08.017
0030-4018/

2. Encoding a single qubit on a spin coherent state

To encode a qubit, we will consider BECs with ground state degrees of freedom, such as two hyperfine levels in an atomic BEC [3]. We assume that temperatures are sufficiently low such that the spatial degrees of freedom are frozen out. Denote the bosonic annihilation operators of the two ground states as a and b. These obey standard bosonic commutation relations $[a,a^\dagger]=[b,b^\dagger]=1$ [15]. We then propose that a standard qubit state $\alpha|0\rangle+\beta|1\rangle$ is now encoded on the BEC in the spin coherent state such that

$$|\alpha,\beta\rangle\rangle \equiv \frac{1}{\sqrt{N!}}(\alpha a^\dagger+\beta b^\dagger)^N|0\rangle, \tag{1}$$

where α and β are arbitrary complex numbers satisfying $|\alpha|^2+|\beta|^2=1$. Double brackets are used to denote spin coherent states, emphasizing the fact that these are macroscopic states involving many particles. We call the state (1) a "BEC qubit" due to the analogous properties of this state with a standard qubit. For simplicity we consider the boson number $N=a^\dagger a+b^\dagger b$ to be a conserved number, which amounts to a zero temperature approximation. Assuming N particles that can be in either level a or b, the Hilbert space has a dimension of $N+1$. Fock states can be written as

$$|k\rangle \equiv \frac{(a^\dagger)^k(b^\dagger)^{N-k}}{\sqrt{k!(N-k)!}}|0\rangle, \tag{2}$$

which are orthonormal $\langle k|k'\rangle=\delta_{kk'}$ with $k\in[0,N]$.

The spin coherent state (1) can be visualized by a point on the Bloch sphere with an angular representation $\alpha=\cos(\theta/2)$, $\beta=\sin(\theta/2)e^{i\phi}$. The spin coherent states form a set of pseudo-orthogonal states for large N. The overlap between two states can be calculated to be

$$\langle\langle\alpha',\beta'|\alpha,\beta\rangle\rangle = e^{-i(\phi-\phi')N/2}\left[\cos\left(\frac{\theta-\theta'}{2}\right)\cos\left(\frac{\phi-\phi'}{2}\right) + i\cos\left(\frac{\theta+\theta'}{2}\right)\sin\left(\frac{\phi-\phi'}{2}\right)\right]^N. \tag{3}$$

For example, for $\phi=\phi'$ the overlap simplifies to

$$\langle\langle\alpha',\beta'|\alpha,\beta\rangle\rangle = \cos^N\left(\frac{\theta-\theta'}{2}\right)\approx\exp\left(-\frac{N(\theta-\theta')^2}{8}\right). \tag{4}$$

Thus beyond angle differences of the order of $\theta-\theta'\sim 1/\sqrt{N}$, the overlap is exponentially suppressed.

The state (1) can be manipulated using total spin (Schwinger boson) operators

$$\begin{aligned} S^x &= a^\dagger b+b^\dagger a,\\ S^y &= -ia^\dagger b+ib^\dagger a,\\ S^z &= a^\dagger a-b^\dagger b, \end{aligned} \tag{5}$$

which satisfy the usual spin commutation relations $[S^i,S^j]=2i\epsilon_{ijk}S^k$, where ϵ_{ijk} is the Levi-Civita antisymmetric tensor. In the spin language, (1) forms a spin-$N/2$ representation of the SU(2) group (we omit the factor of 1/2 in our spin definition for convenience). For the special case of $N=1$, the spin operators reduce to Pauli operators

$$\begin{aligned} \sigma^x &= |1\rangle\langle 0|+|0\rangle\langle 1|,\\ \sigma^y &= -i|1\rangle\langle 0|+i|0\rangle\langle 1|,\\ \sigma^z &= |1\rangle\langle 1|-|0\rangle\langle 0|. \end{aligned} \tag{6}$$

When referring to standard qubits, we will use the $\sigma^{x,y,z}$ notation throughout this paper to differentiate this to the BEC case where we will use $S^{x,y,z}$.

Single BEC qubit rotations can be performed in a completely analogous fashion to regular qubits. For example, rotations around the z-axis of the Bloch sphere can be performed by an evolution

$$\begin{aligned} e^{-i\Omega S^z t}|\alpha,\beta\rangle\rangle &= \frac{1}{\sqrt{N!}}\sum_{k=0}^{N}\binom{N}{k}(\alpha a^\dagger e^{-i\Omega t})^k(\beta b^\dagger e^{i\Omega t})^{N-k}|0\rangle\\ &= |\alpha e^{-i\Omega t},\beta e^{i\Omega t}\rangle\rangle. \end{aligned} \tag{7}$$

Similar rotations may be performed around any axis by an application of

$$H_1=\hbar\Omega\boldsymbol{n}\cdot\boldsymbol{S}=\hbar\Omega(n_xS^x+n_yS^y+n_zS^z) \tag{8}$$

where $\boldsymbol{n}=(n_x,n_y,n_z)$ is a unit vector. Expectation values of the total spin are identical to that of a single spin (up to a factor of N), taking values

$$\begin{aligned} \langle S^x\rangle &= N(\alpha^*\beta+\alpha\beta^*)\\ \langle S^y\rangle &= N(-i\alpha^*\beta+i\alpha\beta^*)\\ \langle S^z\rangle &= N(|\alpha|^2-|\beta|^2), \end{aligned} \tag{9}$$

where $\langle S^{x,y,z}\rangle\equiv\langle\langle\alpha,\beta|S^{x,y,z}|\alpha,\beta\rangle\rangle$. These may be derived efficiently by using the relations

$$\begin{aligned} [S^x,\alpha a^\dagger+\beta b^\dagger] &= \alpha b^\dagger+\beta a^\dagger\\ [S^y,\alpha a^\dagger+\beta b^\dagger] &= -i\alpha b^\dagger+i\beta a^\dagger\\ [S^z,\alpha a^\dagger+\beta b^\dagger] &= \alpha a^\dagger-\beta b^\dagger \end{aligned} \tag{10}$$

and

$$[\alpha^* a+\beta^* b,\alpha a^\dagger+\beta b^\dagger]=1. \tag{11}$$

In contrast to the average spin, when normalized according to $S^{x,y,z}/N$ has the same result as for standard qubits, variance diminishes under the same normalization:

$$\frac{\langle(S^z)^2\rangle-\langle S^z\rangle^2}{N^2}=\frac{4|\alpha\beta|^2}{N}. \tag{12}$$

This is in accordance with the widespread notion that for $N\to\infty$ the spins approach "classical" variables. We shall however see in the following section that despite the classical appearance of such a state, such a many boson state can exhibit quantum properties such as entanglement.

We note that collective state encodings have been proposed previously in works such as Refs. [16–18], where a large number of particles is used to encode a two level system. A key difference between the encoding in these works and (1) is that the full $N+1$ Hilbert space is used here to encode the two level system. Typically in these works first the spins are polarized in a particular direction and low lying excitations are used to encode quantum information. In contrast, for various parameters α,β the state (1) uses the full Hilbert space of the spins. Thus although many physical particles encode the quantum state, the Hilbert space mapping is one-to-one.

3. Entanglement between BECs

Two BEC qubit gates can be formed by any product of the Schwinger boson operators of the form

$$H_2=\sum_{n,m=1}^{M}\sum_{i,j=x,y,z}\hbar\Omega_{ij}S_n^iS_m^j \tag{13}$$

where Ω_{ij} are real symmetric parameters. Our first aim will be to show that such an operator, combined with H_1 allows for a set of operations with the corresponding operations to standard qubit operations. To make this definition more precise, let us consider the most general Hamiltonian for standard qubits:

$$H=\sum_{\boldsymbol{j}}A(\boldsymbol{j})\prod_{n=1}^{M}\sigma_n^{j(n)} \tag{14}$$