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Beam wander of random electromagnetic Gaussian-shell model vortex beams propagating through a Kolmogorov turbulence

Guohua Wu [a,*], Wen Dai [a], Hua Tang [b], Hong Guo [c]

[a] School of Electronic Engineering Beijing University of Posts and Telecommunications, Beijing 100876, China
[b] School of Electronic and Information Engineering, Beihang University, Beijing 100191, China
[c] CREAM Group, State Key Laboratory of Advanced Optical Communication Systems and Networks (Peking University) and Institute of Quantum Electronics, School of Electronics Engineering and Computer Science, Peking University, Beijing, 100871, China

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ABSTRACT

Beam wander of random electromagnetic Gaussian-Shell model (EGSM) vortex beams propagating through atmospheric turbulence is investigated. We develop the expression for beam wander of random EGSM vortex beams in theory. The effects of topological charge, turbulence strength, initial spatially coherent length, transverse scale, and wavelength on beam wander are illustrated numerically. The numerical results show that vortex beams with both positive and negative topological charges have the same beam wander, decreasing the coherent length and decreasing the transverse scale, or increasing the topological charge, can decrease the beam wander. In free-space optical (FSO) communication, we can choose beams with smaller coherent length, smaller wavelength, and larger topological charge to reduce beam wander.

1. Introduction

Recently, FSO communication has drawn considerable attention for its potential high-rate data capacity, high security, high bandwidth, high ability of resistant to electromagnetic interference, and low power requirements [1]. However, atmospheric turbulence causes spatial intensity fluctuations, angle of arrival fluctuations, random changes of beam direction, spatial coherence reducing, additional beam spreading beyond that due to pure diffraction, and random irradiance fluctuations better known as intensity scintillation, which limits the performance of FSO communication system [2–7]. The electromagnetic Gaussian-Shell model (EGSM) beam can significantly reduce the intensity scintillation, which makes it to draw considerable attentions, and it can be characterized by its coherent length and polarization property [8,9]. In addition, vortex beams can also reduce the intensity scintillation [10]. The characteristics of vortex beams are that they have helical structure of wave front, the barrel intensity distribution, the smaller dark pot size and possess certain orbital angular momentum (OAM), which make vortex beams applied in many fields, including FSO communication [11]. To encode OAM of the vortex beams can effectively improve data capacity and transfer rate of the communication system [12]. Nevertheless, due to random distribution of atmospheric refractive index that is the direct consequence of small temperature fluctuations transported by the turbulent motion of the atmosphere, the instantaneous centers of the beams will appear transverse displacement in the receiver plane, producing what is commonly called the beam wander. Beam wander is one of the most important characteristics in determining the system performance, it can be represented statistically by the variance of transverse displacement [13,14].

In the past years, beam wander has been studied widely. In Ref. [15], Berman et al. studied the effect of the initial coherence of the radiation on beam wander. In Ref. [16], Xiao and Voelz studied beam wander of focused partially coherent beams propagating in turbulence. In Ref. [17], Eyyuboglu and Cil studied beam wander of dark hollow, flat-topped and annular beams. In Ref. [18], Cil and Eyyuboglu studied beam wander characteristics of cos and cosh-Gaussian beams. In Ref. [19], Yu and Chen studied the beam wander of the EGSM beams. In Ref. [20,21], Wen and Chu studied the beam wander of Airy beam with a spiral phase and partially coherent Airy beams. The beam wander of random EGSM vortex beams has not been reported. When vortex beams propagating through a turbulent atmosphere, the vortex beams also will be affected by atmospheric turbulence [22–24]. Therefore, it is necessary to study the beam wander of vortex beams.

In this paper, we investigate the beam wander of random EGSM vortex beams and consider the effects of topological charge, turbulence strength, initial spatially coherent lengths, transverse

* Corresponding author.
E-mail address: wuguohua@bupt.edu.cn (G. Wu).

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scale, and wavelength on beam wander. In Section 2, the formulation of beam wander for random EGSM vortex beams propagating through atmospheric turbulence is derived. In Section 3, we give the results of numerical calculation examples and analyse the reasons of this result. In Section 4, we summarize the achievements in this paper.

2. Formulation

The cross-spectral density matrix, which is the Fourier transform of the mutual coherence matrix of random electromagnetic vortex beams at the source plane $z=0$, is expressed as [25]

$$W(\mathbf{r}_1,\mathbf{r}_2,0,\omega)=\begin{bmatrix} W_{xx}(\mathbf{r}_1,\mathbf{r}_2,0,\omega) & W_{xy}(\mathbf{r}_1,\mathbf{r}_2,0,\omega) \\ W_{yx}(\mathbf{r}_1,\mathbf{r}_2,0,\omega) & W_{yy}(\mathbf{r}_1,\mathbf{r}_2,0,\omega)\end{bmatrix}, \tag{1}$$

where

$$W_{ij}(\mathbf{r}_1,\mathbf{r}_2,0,\omega)=\langle E_i^*(\mathbf{r}_1,0,\omega)E_j(\mathbf{r}_2,0,\omega)\rangle \quad (i,j=x,y). \tag{2}$$

In Eq. (2) E_x and E_y are two electric-field components, $*$ and $\langle\cdot\rangle$ denote the complex conjugate and ensemble average, respectively. $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors at the $z=0$ plane, ω is the frequency and the following it can be omitted to simplify. If we assume that the electric field $E_x$and E_y uncorrelated, Eq. (1) can be written as [19,26]

$$W(\mathbf{r}_1,\mathbf{r}_2,0)=\begin{bmatrix} W_{xx}(\mathbf{r}_1,\mathbf{r}_2,0) & 0 \\ 0 & W_{yy}(\mathbf{r}_1,\mathbf{r}_2,0)\end{bmatrix}. \tag{3}$$

The elements in Eq. (3) are expressed as [27]

$$W_{ij}(\mathbf{r}_1,\mathbf{r}_2,0)=A_iA_jB_{ij}[r_{1x}r_{2x}+r_{1y}r_{2y}+i\,\mathrm{sgn}(m)r_{1x}r_{2y} - i\,\mathrm{sgn}(m)r_{2x}r_{1y}]^{|m|}\times\exp\left(-\frac{r_1^2+r_2^2}{\omega_0^2}\right)\times\exp\left[-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{2\delta_{ij}^2}\right], \tag{4}$$

where A_i and A_j are the amplitudes of the electric field-vector components, ω_0 is the transverse scale, δ_{ij} is the coherent length, m is the topological charge, $sgn(m)$ represents the sign function. B_{ij} denotes the correlation coefficient, and

$$B_{ij}=\begin{cases}0 & i\neq j\\ 1 & i=j\end{cases}.$$

In order to calculate conveniently, we take $m=+1$, the cross-spectral density $W_{ij}(\rho_1,\rho_2,z)$ at the receiver plane can be expressed as [27]

$$W_{ij}(\rho_1,\rho_2,z)=A_iA_jB_{ij}\left(\frac{k}{2\pi z}\right)^2\times\exp\left[-\frac{ik}{2z}(\rho_1^2-\rho_2^2)-\frac{1}{\rho_0^2}(\rho_1-\rho_2)^2\right]\times\iint d^2u\iint d^2v\left[\left(u^2-\frac{v^2}{4}\right)-i(u_xv_y-u_yv_x)\right]\times\exp\left[-\frac{2}{\omega_0^2}u_x^2-\frac{2}{\omega_0^2}u_y^2\right]\times\exp\left[-a_{ij}v_x^2-a_{ij}v_y^2\right]\times\exp\left[\frac{ik}{z}(\rho_1-\rho_2)\cdot\mathbf{u}\right]\times\exp\left[-\frac{ik}{z}\mathbf{u}\cdot\mathbf{v}\right]\times\exp\left[\frac{ik}{2z}(\rho_1+\rho_2)\cdot\mathbf{v}\right]\times\exp\left[-\frac{1}{\rho_0^2}(\boldsymbol{\rho}_1-\rho_2)\cdot\mathbf{v}\right], \tag{5}$$

$k=2\pi/\lambda$ is the wavenumber, λ is the wavelength. When $\rho_1=\rho_2=\rho=\sqrt{\rho_x^2+\rho_y^2}$, Eq. (5) reduces to

$$W_{ij}(\rho,z)=A_iA_jB_{ij}\left(\frac{k}{2\pi z}\right)^2\times\iint d^2u\iint d^2v\times\left[\left(u^2-\frac{v^2}{4}\right)-i(u_xv_y-u_yv_x)\right]\times\exp\left[-\frac{2}{\omega_0^2}u_x^2-\frac{2}{\omega_0^2}u_y^2\right]\times\exp\left[-a_{ij}v_x^2-a_{ij}v_y^2\right]\times\exp\left[-\frac{ik}{z}\mathbf{u}\cdot\mathbf{v}\right]\times\exp\left[\frac{ik}{z}\boldsymbol{\rho}\cdot\mathbf{v}\right]. \tag{6}$$

Through calculation, we can find that

$$W_{ij}(\rho,z)=\frac{A_iA_jB_{ij}k^2\omega_0^2}{8z^2f_{ij}}\times\exp\left[-\frac{k^2}{4z^2f_{ij}}\rho^2\right]\times\left[\frac{\omega_0^2}{2}-\frac{k^2\omega_0^4+4z^2}{16z^2f_{ij}}+\frac{k^2(k^2\omega_0^4+4z^2)}{64z^4f_{ij}^2}\rho^2\right] \tag{7}$$

where

$$a_{ij}=\frac{1}{2\omega_0^2}+\frac{1}{2\sigma_{ij}^2}+\frac{1}{\rho_0^2}, \tag{8}$$

$$f_{ij}=a_{ij}+\frac{k^2\omega_0^2}{8z^2}, \tag{9}$$

$\rho_0(z)=(0.55C_n^2k^2z)^{(-3/5)}$ is the coherence length of a spherical wave propagating through turbulence, C_n^2 represents the refractive index structure parameter describing the strength of atmospheric turbulence.

Therefore, the average intensity at the receiver is expressed as

$$I(\rho,z)=W_{xx}(\rho,z)+W_{yy}(\rho,z) =\frac{A_x^2k^2\omega_0^2}{8z^2f_{xx}}\times\exp\left[-\frac{k^2}{4z^2f_{xx}}\rho^2\right]\times\left[\frac{\omega_0^2}{2}-\frac{k^2\omega_0^4+4z^2}{16z^2f_{xx}}+\frac{k^2(k^2\omega_0^4+4z^2)}{64z^4f_{xx}^2}\rho^2\right] +\frac{A_y^2k^2\omega_0^2}{8z^2f_{yy}}\times\exp\left[-\frac{k^2}{4z^2f_{yy}}\rho^2\right]\times\left[\frac{\omega_0^2}{2}-\frac{k^2\omega_0^4+4z^2}{16z^2f_{yy}}+\frac{k^2(k^2\omega_0^4+4z^2)}{64z^4f_{yy}^2}\rho^2\right] \tag{10}$$

In order to calculate conveniently, we can assume that the vortex beams are isotropic. Namely $\delta_{xx}=\delta_{yy}$, then $a_{xx}=a_{yy}=a$, $f_{xx}=f_{yy}=f$. The average intensity can be reduced to

$$I(\rho,z)=\frac{k^2\omega_0^2}{8z^2f}\times\exp\left[-\frac{k^2}{4fz^2}\rho^2\right]\times\left[\frac{\omega_0^2}{2}-\frac{k^2\omega_0^4+4z^2}{16z^2f}+\frac{k^2(k^2\omega_0^4+4z^2)}{64z^4f^2}\rho^2\right]\times(A_x^2+A_y^2) \tag{11}$$

We obtain the long-term beam width $W_{LT}(z)$ in the presence of turbulence, which is the radius of the long-term spot caused by the movement of the short-term beam over a long time period

$$W_{LT}(z)=\left[\frac{2\iint\rho^2I(\rho,z)d^2\rho}{\iint I(\rho,z)d^2\rho}\right]^{1/2}. \tag{12}$$

By substituting Eq. (11) into Eq. (12), we find that

$$W_{LT}(z)=\left[\frac{8\omega_0^2z^2f+k^2\omega_0^4+4z^2}{k^2\omega_0^2}\right]^{1/2}. \tag{13}$$

It shows that the long-term beam width is affected by propagation distance, wavenumber, coherence length, transverse scale and the parameters of turbulence. The beam wander of EGSM vortex beams