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Nonparaxial propagation of a vectorial apertured off-axis anomalous hollow beam

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ABSTRACT

Based on the vectorial Rayleigh–Sommerfeld integral formula and the complex Gaussian expansion of the hard-edge aperture function, an analytical propagation expression for a nonparaxial vectorial off-axis anomalous hollow beam (AHB) passing through a rectangular aperture is derived. The corresponding closed-forms for unapertured case, the far field expression and the scalar paraxial result are also given as special cases of the general formulae. By using the derived formulae, some numerical examples are given to illustrate the nonparaxial propagation properties of a vectorial off-axis AHB through a rectangular aperture. It is shown that the f parameter, the off-axis displacement and the truncation parameter all play an important role in determining nonparaxial propagation behavior.

1. Introduction

Due to the wide applications in modern and atomic optics, conventional dark-hollow beams (DHBs) with zero central intensity have been widely investigated both theoretically and experimentally [1–5]. Various techniques have been used to generate various DHBs, such as the transverse mode selection method, the geometrical optical method, the computer-generated hologram, and spatial filtering, etc. [6–10]. Many different theoretical models have been proposed to describe DHBs, such as the LG modes, dark hollow beams, Bessel–Gaussian beams, higher-order Mathieu beams, hollow Gaussian beams and so on [11–15]. Recently Wu et al. observed an anomalous hollow electron beam of elliptical symmetry with an elliptical solid core in experiments it can be used as a powerful tool for studying the linear and nonlinear particle dynamics in the storage ring [16]. More recently, Cai first proposed two convenient theoretical models to describe the anomalous hollow beam (AHB) [17,18]. Since then, within the paraxial framework, the propagation properties of coherent and partially coherent AHB under various cases have been widely studied [19–24]. However, the paraxial approximation is no longer valid when the far-field divergent angle becomes large or when the beam spot size and wavelength are comparable. By using the vector angular spectrum representation, the characteristic of a nonparaxial AHB in the far fields has been examined from the vectorial structure [25]. But, in practice case, the hard apertures always exist in most optical systems. Moreover, the general case of off-axis propagation should be considered. Therefore, the aim of the present paper is to study the nonparaxial propagation of a vectorial off-axis AHB through a rectangular aperture.

The present paper is organized as follows. In Section 2, based on the vectorial Rayleigh–Sommerfeld integrals formula and the complex Gaussian expansion of the hard-edge aperture function, an analytical propagation expression of a nonparaxial vectorial apertured off-axis AHB is derived. In Section 3, some numerical results and analyses are given. Finally, the main conclusions obtained from the present paper are presented in Section 4.

2. Nonparaxial propagation of a vectorial apertured off-axis anomalous hollow beam

In the Cartesian coordinate system consider an AHB linearly polarized in the x direction, whose field $E=E_x(x_0,y_0,0)\boldsymbol{i}+E_y(x_0,y_0,0)\boldsymbol{j}$ at the source plane; $\boldsymbol{i}$ and $\boldsymbol{j}$ denote unit vectors in the x and y directions, respectively. The z-axis is taken to be the propagation axis, a vectorial off-axis AHB in the source plane $z=0$ is described by [17]

$$\begin{pmatrix} E_{0x}(x_0,y_0,0) \\ E_{0y}(x_0,y_0,0) \end{pmatrix} = \begin{pmatrix} E_0\left[-2+\frac{8(x_0-x_d)^2}{w_{0x}^2}+\frac{8(y_0-y_d)^2}{w_{0y}^2}\right]\exp\left[-\frac{(x_0-x_d)^2}{w_{0x}^2}-\frac{(y_0-y_d)^2}{w_{0y}^2}\right] \\ 0 \end{pmatrix}, \tag{1}$$

where E_0 is a normalized constant, w_{0x} and w_{0y} are the beam waist size of an astigmatic Gaussian beam in x and y directions, and

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x_d and y_d are the off-axis displacements in the x and y directions, respectively. The time dependent factor $\exp(-i\omega t)$ is omitted in Eq. (1), here ω is the circular frequency. Assume that a rectangular aperture is positioned in the source plane. The center of the rectangular aperture coincides with the origin of the Cartesian coordinate system. The widths of the rectangular aperture in x and y directions are $2a$ and $2b$, respectively. The optical field just behind the rectangular aperture is expressed as

$$\begin{pmatrix} E_x(x_0,y_0,0) \\ E_y(x_0,y_0,0) \end{pmatrix} = \mathrm{rect}\,(x_0,y_0) \times \begin{pmatrix} E_0\left[-2+\frac{8(x_0-x_d)^2}{w_{0x}^2}+\frac{8(y_0-y_d)^2}{w_{0y}^2}\right]\exp\left[-\frac{(x_0-x_d)^2}{w_{0x}^2}-\frac{(y_0-y_d)^2}{w_{0y}^2}\right] \\ 0 \end{pmatrix}, \tag{2}$$

where $\mathrm{rect}(x_0,y_0)$ is the hard-edge rectangular aperture function given by

$$\mathrm{rect}(x_0,y_0) = \begin{cases} 1, |x_0|<a, |x_0|<b, \\ 0, |x_0|>a, |x_0|>b. \end{cases} \tag{3}$$

The hard-edge rectangular aperture function can be expanded into a finite sum of complex Gaussian functions as follows [26-28]

$$\mathrm{rect}\,(x_0,y_0) = \sum_{m=1}^{N} A_m \exp(-\frac{B_m}{a^2}x_0^2) \sum_{n=1}^{N} A_n \exp(-\frac{B_n}{b^2}y_0^2), \tag{4}$$

where the complex-expansion Gaussian coefficients A_m, A_n, B_m and B_n can be obtained by optimization computation directly; a table of A_m and B_m, or A_n and B_n, can be found in [26]. This expansion method has been proved to be reliable and efficient. The simulation accuracy improves as N increases. For a hard aperture, $N=10$ assures a very good agreement with the straightforward diffraction integration [26], so we take $N=10$ in the following numerical calculations. By using the vectorial Rayleig–Sommerdeld integral formula, the nonparaxial vectorial apertured off-axis AHB propagating toward half free space $z>0$ is found to be [29]

$$E_x(r) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_x(x_0,y_0,0)\frac{\partial G(r,r_0)}{\partial z}dx_0\,dy_0, \tag{5}$$

$$E_y(r) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_y(x_0,y_0,0)\frac{\partial G(r,r_0)}{\partial z}dx_0\,dy_0, \tag{6}$$

$$E_z(r) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[E_x(x_0,y_0,0)\frac{\partial G(r,r_0)}{\partial x}+E_y(x_0,y_0,0)\frac{\partial G(r,r_0)}{\partial y}\right]dx_0\,dy_0, \tag{7}$$

where

$$G(r,r_0) = \frac{\exp(ik|\boldsymbol{r}-\boldsymbol{r}_0|)}{|\boldsymbol{r}-\boldsymbol{r}_0|}, \tag{8}$$

$\boldsymbol{r}=x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}$ and $\boldsymbol{r}_0=x_0\boldsymbol{i}+y_0\boldsymbol{j}$ with $\boldsymbol{i},\boldsymbol{j}$ and $\boldsymbol{k}$ being units vectors along x-, y- and z-axes in the Cartesian coordinate system, respectively, and $k=2\pi/\lambda$ is the wave number with λ being the incident wavelength. Beyond the paraxial approximation, the following approximation is valid:

$$G(r,r_0) = \frac{1}{r}\exp\left[ik\left(r+\frac{x_0^2+y_0^2-2xx_0-2yy_0}{2r}\right)\right], \tag{9}$$

where $r=(x^2+y^2+z^2)^{1/2}$. Substituting Eqs. (2), (4) and (9) into Eqs. (5)–(7), after tedious but straightforward integration, we can obtain the following analytical expressions of a vectorial apertured off-axis AHB beyond paraxial approximation:

$$E_x(x,y,z) = -\frac{ikzE_0}{r^2}\exp\left(ikr-\frac{x_d^2}{w_{0x}^2}-\frac{y_d^2}{w_{0y}^2}\right)\sum_{m=1}^{N}\sum_{n=1}^{N}\frac{A_mA_n}{\sqrt{C_xC_y}}\exp\left(\frac{D_x^2}{4C_x}+\frac{D_y^2}{4C_y}\right) \times\left\{-1+\frac{1}{w_{0x}^2}\left[\left(\frac{D_x}{C_x}-2x_d\right)^2+\frac{2}{C_x}\right]+\frac{1}{w_{0y}^2}\left[\left(\frac{D_y}{C_y}-2y_d\right)^2+\frac{2}{C_y}\right]\right\}, \tag{10}$$

$$E_y(x,y,z)=0, \tag{11}$$

$$E_z(x,y,z) = \frac{ikE_0}{r^2}\exp\left(ikr-\frac{x_d^2}{w_{0x}^2}-\frac{y_d^2}{w_{0y}^2}\right)\sum_{m=1}^{N}\sum_{n=1}^{N}\frac{A_mA_n}{\sqrt{C_xC_y}}\exp\left(\frac{D_x^2}{4C_x}+\frac{D_y^2}{4C_y}\right) \times\left\langle x\left\{-1+\frac{1}{w_{0x}^2}\left[\left(\frac{D_x}{C_x}-2x_d\right)^2+\frac{2}{C_x}\right]+\frac{1}{w_{0y}^2}\left[\left(\frac{D_y}{C_y}-2y_d\right)^2+\frac{2}{C_y}\right]\right\} -\frac{D_x}{2C_x}\left\{-1+\frac{1}{w_{0x}^2}\left[\left(\frac{D_x}{C_x}-2x_d\right)^2+\frac{6}{C_x}-\frac{8x_d}{D_x}\right]+\frac{1}{w_{0y}^2}\left[\left(\frac{D_y}{C_y}-2y_d\right)^2+\frac{2}{C_y}\right]\right\}\right\rangle, \tag{12}$$

where

$$C_x = \frac{B_m}{a^2}+\frac{1}{w_{0x}^2}-\frac{ik}{2r}, \tag{13}$$

$$C_y = \frac{B_n}{b^2}+\frac{1}{w_{0y}^2}-\frac{ik}{2r}, \tag{14}$$

$$D_x = \frac{2x_d}{w_{0x}^2}-\frac{ikx}{r}, \tag{15}$$

$$D_y = \frac{2y_d}{w_{0y}^2}-\frac{iky}{r}, \tag{16}$$

Eqs. (10)–(12) are the main results obtained in this paper, which provide a general propagation expression of a vectorial apertured off-axis AHB beyond paraxial approximation. When$a\to\infty$and $b\to\infty$ in Eqs. (13) and (14), which means the case where there exists no aperture, then Eqs. (10)–(12) are just the propagation expressions for a vectorial nonparaxial off-axis AHB in free space.

The far field and paraxial expressions of a vectorial apertured off-axis AHB can also be obtained under the far field and paraxial approximation conditions. In the far field regime, Eq. (9) can be simplified into the following form:

$$G(r,r_0) = \frac{1}{r}\exp\left[ik\left(r+\frac{-2xx_0-2yy_0}{2r}\right)\right]. \tag{17}$$

Accordingly, the far field expressions of a nonparaxial vectorial apertured off-axis AHB are reduced into the following forms:

$$E_{xf}(x,y,z) = -\frac{ikzE_0}{r^2}\exp\left(ikr-k^2f_x^2x_d^2-k^2f_y^2y_d^2\right)\sum_{m=1}^{N}\sum_{n=1}^{N}\frac{A_mA_n}{\sqrt{G_xG_y}} \times\exp\left(\frac{{H_x}^2}{4G_x}+\frac{{H_y}^2}{4G_y}\right)\times\left\{-1+k^2f_x^2\left[\left(\frac{H_x}{G_x}-2x_d\right)^2+\frac{2}{G_x}\right] +k^2f_y^2\left[\left(\frac{H_y}{G_y}-2y_d\right)^2+\frac{2}{G_y}\right]\right\}, \tag{18}$$

$$E_{yf}(x,y,z)=0, \tag{19}$$

$$E_{zf}(x,y,z) = \frac{ikE_0}{r^2}\exp\left(ikr-k^2f_x^2x_d^2-k^2f_y^2y_d^2\right)\sum_{m=1}^{N}\sum_{n=1}^{N}\frac{A_mA_n}{\sqrt{G_xG_y}}\exp\left(\frac{H_x^2}{4G_x}+\frac{H_y^2}{4G_y}\right) \times\left\langle x\left\{-1+k^2f_x^2\left[\left(\frac{H_x}{G_x}-2x_d\right)^2+\frac{2}{G_x}\right]+k^2f_y^2\left[\left(\frac{H_y}{G_y}-2y_d\right)^2+\frac{2}{G_y}\right]\right\} -\frac{H_x}{2G_x}\left\{-1+k^2f_x^2\left[\left(\frac{H_x}{G_x}-2x_d\right)^2+\frac{6}{G_x}-\frac{8x_d}{H_x}\right] +k^2f_y^2\left[\left(\frac{H_y}{G_y}-2y_d\right)^2+\frac{2}{G_y}\right]\right\}\right\rangle, \tag{20}$$

where

$$f_x=1/kw_{0x}, f_y=1/kw_{0y}, \delta_x=a/w_{0x}, \delta_y=a/w_{0y} \tag{21}$$