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Orbital angular momentum and highly efficient holographic generation of nondiffractive TE and TM vector beams

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ABSTRACT

This paper discusses the angular momentum to energy ratio for a class of nondiffractive vector beams. Using the Humblet decomposition, we introduce closed form equations for the orbital, the spin, and the surface angular momenta in both paraxial and nonparaxial regimes. The considered monochromatic beams are exact solution to the Maxwell equations in free space and can be either transverse electric (TE) or transverse magnetic (TM). In this context, we analytically show that the total angular momentum is purely orbital. Additionally, we address both numerically and experimentally the generation of nondiffractive vector beams. In the generation of the vector beams, we propose a general approach to encode the corresponding scalar beams into the Kinoform, which possesses the upper bound diffraction efficiency. Our approach is general in the sense that we can encode arbitrary nondiffractive TE and TM vector beams. The experimental setup consists of two stages; a 4-f system and a common path interferometer. To highlight the proposed approach, we experimentally generate high efficiency Bessel, Mathieu, and Weber vector beams.

1. Introduction

Vector beams have attracted significant interest due to their unique properties, e.g., the presence of a strong longitudinal component of the electric field, tighter focusing properties, spin momentum, and orbital angular momentum [1–3]. In particular, vector beams that possess orbital angular momentum are useful in many practical applications such as optical tweezers, imaging, and wireless communications [3–5].

Homogenously polarized vector beams may carry orbital angular momentum and spin momentum. In the paraxial case, the spin part depends on the polarization whereas the orbital part relies on the magnitude and phase distributions. In addition, the orbital angular momentum is polarization free [6–8]. Linearly and circularly polarized Laguerre–Gauss beams possess well-defined angular momentum, which can be separated into the orbital and spin parts [6]. Later, in [7], the authors found similar expressions for elliptically polarized beams, where the considered beams have azimuthal phase dependence like Laguerre–Gauss beams. The numerical computation of the orbital angular momentum for Mathieu beams was considered in [8]. Similarly, for Bessel beams, the analytical computation of the orbital angular momentum is derived in [4].

Further investigations for a general nonparaxial form of the orbital angular momentum in homogenously polarized vector beams are, for example, given in [4,7,9]. In [7], the authors considered vector beams with azimuthal phase dependence like Laguerre–Gauss beams. They found that no simple separation of the total angular momentum into the orbital angular momentum and spin momentum exists. However, the relationship for the total angular momentum in the paraxial and nonparaxial regimes is the same. In addition, the case of vector Bessel beams was considered in [4]. The work [9] considers the computation of the orbital angular momentum using a general equation for the vector beams, that is, the angular plane wave spectrum of the nonparaxial beam. That approach involves the use of Fourier series decomposition to calculate the orbital angular momentum.

In nonhomogenously polarized vector beams, radially and azimuthally polarized Bessel vector beams were considered in [10]. This approach is based on the Hertz vector potential and the resulting vector beams are solution of Maxwell's equations. In addition, the author showed that the total angular momentum equals zero. Similarly, in [11], the authors proposed an interesting class of monochromatic transverse electric (TE) and transverse magnetic (TM) nondiffractive vector beam modes, which are exact solution of Maxwell's equations in free space. Using this approach, Bessel, Mathieu, and Weber vector fields with different

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polarization states can be obtained. Radial and azimuthal polarizations are special cases [4,11]. Additionally, the authors provided the energy density and the Pointing vector of these beams. Unfortunately, closed form equations for the evaluation of the orbital angular momentum and spin momentum exist for some special cases, that is, low order Bessel beams [4,12,13].

In the aforementioned nonparaxial vector beams, the total angular momentum accounts for the spin and orbital parts. Recently, however, it was suggested that in order to preserve the gauge invariance of the beam, we should include an extra term called surface angular momentum [14]. In this fashion, the surface part is not negligible in the nonparaxial regime for Laguerre–Gaussian and Bessel beams [14].

On the other hand, in the context of digital holography, several methods for generating vector beams have been proposed in the literature [12,15–19]. One promising method is the use of spatial light modulator (SLM) due to its versatility and flexibility [17–19]. In this setting, the SLM transforms the incoming beam into two previously designed and homogenously orthogonally polarized scalar beams, which are superimposed to generate the corresponding vector beam. Hence, using amplitude computer generated holograms and an axicon, the generation of nondiffractive first-order TE and TM Bessel vector beams in free space was proposed [12]. In [17], it was suggested the generation of vector beams using ferroelectric liquid crystal SLM, which diffracts the two incoming orthogonal polarized beams with equal efficiency. Unfortunately, the proposed SLM displays binary diffractive structures and the resulting beams have poor diffraction efficiency. In [18], the authors proposed an interesting method to produce arbitrary vector beams using amplitude SLM and a common path interferometric arrangement. Although they generate stable and good quality vector beams, the use of amplitude SLM results in poor diffraction efficiency. To generate high quality vector beams and high diffraction efficiency, a convenient way is the use of phase holograms. Accordingly, the author in [19] showed that phase holograms allow to control the superposition of scalar beams of different orders and the resulting vector beams possess high signal to noise ratio (SNR). However, the encoding method used to generate the corresponding scalar beams results in low diffraction efficiency which is not suitable for many applications like particle manipulation [20].

The aim of this paper is two fold. First, we provide closed form equations to compute the total angular momentum to energy ratio for a class of nondiffractive TE and TM vector beams. Specifically, we introduce closed form equations for the spin, the orbital, and the surface parts of the angular momentum. Second, we experimentally demonstrate the efficient generation of TE and TM vector beams using phase holograms that possess the upper bound diffraction efficiency. The phase holograms are designed such that they transform the incident beam into the desired scalar beams. Additionally, the proposed phase holograms are implemented in a phase-only SLM, which is placed at the input of a 4-f system. The generated scalar beams are orthogonally polarized and are superposed in a common path interferometric arrangement. To highlight the proposed approach, we experimentally generate high efficiency Bessel, Mathieu, and Weber vector beams.

2. Theory

This section reviews the theory of nondiffractive TE and TM vector beams and introduces closed form equations for the total angular momentum to energy ratio along the propagation axis. For convenience, we use cylindrical coordinates and cartesian coordinates if necessary.

At first, we define the TE and TM electric fields in which we are interested. Following the ideas in [11] and assuming harmonic time dependence of the form $\exp(-i\omega t)$, where ω is the angular frequency of the beam, the class of monochromatic fields that are exact solution to the Maxwell equations in free space is defined as

$$\mathbf{E}^{\mathrm{TE}}(r,\theta,z)=\sqrt{2}[\psi_{+}(r,\theta)\mathbf{e}_{-}-\psi_{-}(r,\theta)\mathbf{e}_{+}]\exp(ik_z z), \tag{1}$$

$$\mathbf{E}^{\mathrm{TM}}(r,\theta,z)=\frac{k_z}{k}\left\{-\sqrt{2}[\psi_{+}(r,\theta)\mathbf{e}_{-}+\psi_{-}(r,\theta)\mathbf{e}_{+}]+2\frac{k_t^2}{k_z}\psi(r,\theta)\mathbf{e}_z\right\}\exp(ik_z z), \tag{2}$$

where (r,θ) are the cylindrical coordinates; k_t and k_z are, respectively, the transverse and longitudinal components of the propagation vector, and are related by the wave number k as $k^2=k_t^2+k_z^2$; the set $\{\mathbf{e}_{+},\mathbf{e}_{-},\mathbf{e}_z\}$ is the cylindrical circular polarization basis; and the nondiffractive scalar beams $\psi_{\pm}(r,\theta)$ are given by [11]

$$\psi_{\pm}(r,\theta)=\frac{1}{i}\exp(\pm i\theta)\left(\frac{\partial}{\partial r}\pm i\frac{1}{r}\frac{\partial}{\partial\theta}\right)\psi(r,\theta), \tag{3}$$

where $\psi(r,\theta)$ is an arbitrary nondiffractive scalar beam. As a difference with the results given in [11], here we propose the use of the Wittaker integral to express a more general form of the nondiffractive scalar beam $\psi(r,\theta)$ [21], i.e.,

$$\psi(r,\theta)=\frac{1}{k_t}\int_{-\pi}^{\pi}A(\varphi)\exp[ik_t r\cos(\varphi-\theta)]\,\mathrm{d}\varphi. \tag{4}$$

The expression for the beam $\psi(r,\theta)$ can be viewed as the inverse Fourier transform in cylindrical coordinates of $\Psi(\rho,\varphi)=(2\pi)^2A(\varphi)\delta(\rho-k_t)/k_t\rho$, where $\delta(\cdot)$ is the Dirac delta, and $A(\varphi)$ is the angular spectrum of the beam. The selection of $A(\varphi)$ depends on the desired intensity distribution of $\psi(r,\theta)$. The well known Bessel, Mathieu, and Weber beams are obtained if $A(\varphi)=\exp(in\varphi)$; $A(\varphi)=ce_m(\varphi;q)+ise_m(\varphi;q)$, where $ce_m(\varphi;q)$ and $se_m(\varphi;q)$ are the m-th order angular Mathieu functions with ellipticity parameter q; and $A(\varphi)=\tan^{i\mu}(\varphi/2)H(\sin\varphi)/\sqrt{\sin\varphi}$, where μ is a real valued parameter, and $H(\cdot)$ is the Heaviside function [22–25].

Substituting (4) into (3), the following relation holds:

$$\psi_{\pm}(r,\theta)=\int_{-\pi}^{\pi}\exp(\pm i\varphi)A(\varphi)\exp[ik_t r\cos(\varphi-\theta)]\,\mathrm{d}\varphi. \tag{5}$$

Expressing the orthonormal circular polarization basis $\mathbf{e}_{\pm}$ in terms of the cylindrical basis, i.e., $\mathbf{e}_{\pm}=(\mathbf{e}_r\pm i\mathbf{e}_\theta)\exp(\pm i\theta)/\sqrt{2}$, Eqs. (1) and (2) become

$$\begin{aligned}\mathbf{E}^{\mathrm{TE}}(r,\theta,z)=\{&[\psi_{+}(r,\theta)\exp(-i\theta)-\psi_{-}(r,\theta)\exp(i\theta)]\mathbf{e}_r\\&-i[\psi_{+}(r,\theta)\exp(-i\theta)+\psi_{-}(r,\theta)\exp(i\theta)]\mathbf{e}_\theta\}\exp(ik_z z),\end{aligned} \tag{6}$$

$$\begin{aligned}\mathbf{E}^{\mathrm{TM}}(r,\theta,z)=\frac{1}{k}(k_z\{&i[\psi_{+}(r,\theta)\exp(-i\theta)-\psi_{-}(r,\theta)\exp(i\theta)]\mathbf{e}_\theta\\&-[\psi_{+}(r,\theta)\exp(-i\theta)+\psi_{-}(r,\theta)\exp(i\theta)]\mathbf{e}_r\}\\&+2k_t^2\psi(r,\theta)\mathbf{e}_z)\exp(ik_z z),\end{aligned} \tag{7}$$

Similarly, using orthonormal cartesian basis, $\mathbf{e}_{\pm}=(\mathbf{e}_x\pm i\mathbf{e}_y)/\sqrt{2}$, we have

$$\mathbf{E}^{\mathrm{TE}}(r,\theta,z)=\{[\psi_{+}(r,\theta)-\psi_{-}(r,\theta)]\mathbf{e}_x-i[\psi_{+}(r,\theta)+\psi_{-}(r,\theta)]\mathbf{e}_y\}\exp(ik_z z), \tag{8}$$

$$\begin{aligned}\mathbf{E}^{\mathrm{TM}}(r,\theta,z)=\frac{1}{k}(k_z\{&i[\psi_{+}(r,\theta)-\psi_{-}(r,\theta)]\mathbf{e}_y-[\psi_{+}(r,\theta)+\psi_{-}(r,\theta)]\mathbf{e}_x\}\\&+2k_t^2\psi(r,\theta)\mathbf{e}_z)\exp(ik_z z),\end{aligned} \tag{9}$$

We now focus our attention to the total angular momentum to energy ratio. So, we introduce closed form equations to compute the orbital angular momentum, the spin momentum, and surface angular momentum for the TE and TM modes. The cycle-averaged angular momentum density of light is expressed as