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Influence of graded index materials on the photonic localization in one-dimensional quasiperiodic (Thue–Mosre and Double-Periodic) photonic crystals

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ABSTRACT

In this paper, we present the investigation on the photonic localization and band gaps in quasi-periodic photonic crystals containing graded index materials using a transfer matrix method in region 150–750 THz of the electromagnetic spectrum. The graded layers have a space dispersive refractive index, which vary in a linear and exponential fashion as a function of the depth of layer. The considered quasiperiodic structures are taken in the form of Thue–Morse and Double-Periodic sequences. The grading profile in the layers affects the position of reflection dips and forbidden bands, and frequency region of the bands. We observed that vast number of forbidden band gaps and dips are developed in its reflection spectra by increasing the number of quasi-periodic generation. Moreover, we compare the total forbidden bandwidths with increasing the generation of the quasi-periodic sequences for the structures with linear and exponential graded layer. Results show that the different graded profiles with same boundary refractive index can change the position of localization modes, number of photonic bands and change the frequency region of the bands. Therefore, we can achieve suitable photonic band gaps and modes by choosing the different gradation profiles of the refractive index and generation of the quasi-periodic sequences.

1. Introduction

In the past two decades, great efforts have been dedicated towards the investigation of the structure and physical properties of quasi-periodic systems after the discovery of the quasi-crystalline structure [1–3]. Quasi-periodic systems have long-range order but not in a repeating fashion yielding periodicity. These arrangements are fixed in a regular pattern and follow a simple deterministic recursion rule [4,5]. Quasi-periodic systems are one of the most interesting arrangements to obtain the suitable photonic band gaps because of several structural parameters available to tune as compared to the periodic and disordered systems [6–8]. Recently, some research groups have reported their works on electromagnetic (EM) wave propagation in quasi-periodic structures called photonic quasi-crystals. Due to a long-range order, such type of structures provide wide photonic band gap in photonic spectra as in periodic photonic crystals and simultaneously possess localized states as in disorder media [9,10]. Photonic quasi-crystals exhibit unique influence on the optical properties such as optical transmission and reflectivity, photoluminescence, light transport, plasmonics and laser action, etc. Li et al. [11] and Luo et al. [12] proposed two-dimensional photonic crystals that achieve multimode lasing action, low pumping threshold and excellent linear polarization property as well as wide directional dependence. This opens a new field of research in photonics in view of their vast technical applications. Photonic band gap properties of quasi-periodic multi-layered structures have been extensively studied for different materials [13]. Specifically, one-dimensional (1-D) photonic quasi-crystals are very important because their formation is relatively easy and they may provide the description of light propagation in one direction [14–16]. One-dimensional photonic quasi-crystals are composed of layers according to substitutional sequences in form of the Fibonacci, Thue–Morse and Double-Periodic etc.

Recently several researchers have been proposed the 1-D multilayer structures with gradual varying RI as a function of the depth of layer, and width of layers varies as a gradual fashion along the direction perpendicular to the surface of layer in the considered structures [17–21]. Such type of structures are called graded photonic crystals (GPCs). In two-dimensional GPC structures, gradual variations of the relative parameters can be distributed along the normal or perpendicular to the propagation of electromagnetic waves. Gradual variation of relative parameters of GPCs make them very different in the behaviour from the conventional PCs and enhance the ability to mold and control of the light wave propagation [22–24]. Such types of PCs play an important role to

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design spectral filters, beam aperture and deflector, high efficiency bending waveguides, high efficiency couplers, super bending and self-focusing media, lenses, artificial optical black holes and antireflection coating [25–30]. In our previous work [31], we have formulated the resonant Bragg condition for the quasi-periodic Fibonacci multilayer structures containing exponential graded material and shown that forbidden band gaps and omni-directional band gaps to be obtained for periodicity of different generation Fibonacci sequence structures and their hetrostructures. In these structures, bandwidth of forbidden and omni-directional band gaps can be tuned with graded profile parameter of exponential graded layer.

Motivated by the ability to mold, confine and control of the electromagnetic waves by different types of GPCs, we now present the study of the photonic band gap characteristics of 1-D quasi-periodic GPCs constituted with an exponential/linear graded dielectric layer. The aims of this work are, we want to show the reflection spectra, which arise from the propagation of electromagnetic waves in quasi-periodic multilayer structures, comprised of alternating layers of both normal (SiO_2) and graded index materials using a theoretical model based on a transfer matrix treatment. The quasi-periodic structures follow the Thue–Morse (TM) and Double-Periodic (DP) substitutional sequences and can be generated by the following inflation rules: A→AB, B→BA (TM); and A→AB, B→AA (DP),[4,5] where A and B are the building blocks modelling of the normal and graded index materials, respectively. Further, we intend to present a quantitative analysis of the results, pointing out the distribution of the forbidden band gaps and total bandwidths for up to the 6th generation, which gives a good insight about their photonic band gaps and localization.

The plan of this paper is as follows. In Section 2, we present the method of calculation employed here, which is based on the transfer matrix approach. The reflection coefficient and dispersion relation is then determined. Section 3 is devoted to the discussion of this reflection spectra and dispersion relation for the Thue–Morse and Double-Periodic multilayer structures containing linear and exponential graded index material. Further, we present their total band gap with Thue–Morse and Double-Periodic generations for linear and exponential graded index material as one of the layer. The conclusions of this work are presented in Section 4.

2. Theoretical model and numerical analysis

In this paper, we consider the system of multilayer that is composed of two layers and stacked alternatively along the x-direction. The stacks of two layers are arranged according to the recursion rule of the Thue–Morse (TM) and Double-Periodic (DP) sequence in different generations [4,5]. These sequences are based on the two-letter alphabet (A, B) and the substitution rule: σ (A)=AB, σ (B)=BA (TM); and σ (A)=AB, σ (B)=AA (DP). The substitution rule can be written in the form of the following equations:

$$\sigma : \begin{pmatrix} A \\ B \end{pmatrix} \to \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} AB \\ BA \end{pmatrix} \text{ and}$$

$$\sigma : \begin{pmatrix} A \\ B \end{pmatrix} \to \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} AB \\ AA \end{pmatrix}$$

The ratio of the frequencies of the letters A, B in the sequence is equal for TM and 2 to 1 for DP. The length of the sequence in both cases at the iteration n is 2^n. On the basis of the above substitution rule, the first few words generated in this way are represented in Table 1 panels (i) and (ii).

The proposed multilayer structures consist of two kinds of layers; one has a constant refractive index (1.5) and other has a linear or exponential varying refractive index (1.5–4.1) as the case may be. The variation of refractive index in the graded layers is taken along the direction of the thickness of the layer. The direction of wave propagation is considered along the x-axis i.e. the direction normal to the stacked layers and the considered materials assumed as non-magnetic, non-dispersive and isotropic. The refractive indices of the considered graded layers vary in linear and exponential fashion between the initial and final values as $[n_L(x) = n_i + ((n_f - n_i)/d_L)x]$ and $[n_E(x) = n_i \exp(x/d_E \log n_f/n_i)]$, respectively. Here, n_i and n_f are the refractive indices at inward and outward boundaries of the graded layer, respectively [32].

The optical properties of the considered multi-layered structures are described by well-known theoretical model based on transfer matrix method (TMM). Transfer matrices are generated by applying boundary conditions in a plane wave solution of the Maxwell's wave equation at the interface boundary. The electric field distribution $E(x)$ in different materials can be written as:

(i) For normal layers:

$$E_N(x) = A_N \exp(-i \cdot k_N \cdot x) + B_N \exp(i \cdot k_N \cdot x)\ldots\ldots\ldots \quad (1)$$

where A_N and B_N are arbitrary constants and $k_N = \omega \cdot n_N / c$ represents the propagation wave vector at normal incidence with a constant refractive index n_N. Subscript N represents the normal layer and ω and c are the angular frequency and velocity of light, respectively [30].

(ii) For linear graded layers the electric field equation can be written as

$$E_L(x) = \sqrt{\xi_L} \cdot \left[A_L J_{1/4}\left(\frac{\xi_L^2}{2\alpha}\right) + B_L Y_{1/4}\left(\frac{\xi_L^2}{2\alpha}\right) \right] \quad (2)$$

where, A_L and B_L are arbitrary constants and $\xi_L = \omega \cdot n_L(x)/c$ the propagation wave vector at normal incidence along x-direction for the linear graded layer with refractive index: $n_L(x) = n_i + ((n_f - n_i)/d_L)x$, where n_i and n_f are the refractive indices at the initial and final boundary and d_L is the layer thickness. Subscript L represents the linear graded layer and grading profile parameter of the linearly graded layer is $\alpha = (\omega/c)(n_f - n_i/d_L)$, ω and c are the angular frequency and velocity of light, respectively [32].

(iii) For exponential graded layers:

$$E_E(x) = A_E J_0\left(\frac{\xi_E}{\gamma}\right) + B_E Y_0\left(\frac{\xi_E}{\gamma}\right)\ldots\ldots\ldots \quad (3)$$

where, A_E and B_E are arbitrary constants and $\xi_E = \omega.n_E(x)/c$ represents the wave propagation vector at normal incidence along the x-direction for the exponential graded layer with refractive index $n_E(x) = n_i \exp(\gamma x)$, where $\gamma = 1/d_E \log(n_f/n_i)$, is the grading profile parameter of the exponentially graded layer, n_i and n_f are same as defined in Eq. (2) and d_E is the thickness of the exponentially graded layer. Subscript E represents the exponentially graded layer. The functions J and Y are the first and second kind Bessel's functions, respectively.

Using the transfer matrix approach, the amplitudes A_0 and B_0 of the electromagnetic field in the air medium at $x < 0$ related to be the amplitudes A_{n+1} and B_{n+1} of the equivalent layer in the $(n+1)$ th region through the linear transformation. Therefore, for the multilayer structures, the total transfer matrix equation can be written as

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = M_{i,j} \begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} \quad (4)$$