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# Optimal bipartite entanglement transfer and photonic implementations

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ABSTRACT

Using local operations and classical communication two remote participants may transform one shared entanglement to other form. In this paper, we evaluate its success probability for arbitrary initial entanglement and final entanglement by using the majorization condition and solving an equivalent nonlinear optimization problem. The optimal probability is determined by their entanglement coefficients. The theoretical scheme may be approximated by an adaptable and iterative scheme, and is schematically realized using the phonotic entanglement with the help of the cross-Kerr nonlinearity.


## 1. Introduction

Different from classical resources, quantum entanglement has been explored to improve the manipulation and transmission of information in various quantum applications [1,2]. Based on the nonlocality, entanglement is used for quantum teleportation [3,4], quantum dense coding [5,6], quantum secret sharing [7,8], quantum secure direct communication [9,10], and so on. These schemes mostly depend on qubit systems. As extensions, $d$-dimensional quantum states (qudits) and $d$-dimensional entangled systems have attracted great attentions because of their larger nonlocality and powerful capability in quantum information processing. In fact, the larger systems may provide different quantum correlations and can be used to improve the security of quantum cryptography schemes [11–15], the implementation of quantum logic gates [16,17], and the experimental simulation of quantum algorithms [18]. These applications are mostly based on special entanglements such as the maximal entanglement. Unfortunately, experimentally prepared entanglements are always unsuitable for these quantum tasks because of unavoidable noise environments or imperfect experiments. Moreover, some entanglements such as photonic entanglements have to be stored in special quantum repeaters for long-distance quantum communication and may be affected by quantum decoherence. Therefore, how to obtain required entanglement from other entanglements is worth investigating for quantum applications.

In this paper, we only focus on the bipartite entanglement transfer under the local operations and classical communications (LOCC). Bennett et al. [19] firstly introduced a special bipartite entanglement transfer, i.e., entanglement concentration protocol (ECP) in which two participants change partially entangled Bell states into the maximal entanglement using the Schmidt projection method and collective measurements for two-photon systems. Here, the entanglement is known to two participants. Nielsen [20] presented an equivalent majorization condition for the deterministic entanglement transfer. For arbitrary initial partial entanglement, the optimal success probability is obtained by Lo and Popescu [21]. Followed these results, various schemes have been proposed for the theoretical or experimental qubit entanglement concentration [22–40], or the qudit entanglement concentration [41,42]. However, few result has been obtained for general case [20,43], i.e., transforming arbitrary entangled qudit to other entanglement. Here, we firstly investigate the success probability of general bipartite entanglement transfer by defining new entanglement in terms of the majorization condition and solving an equivalent nonlinear optimization problem. The optimal probability is determined by the minimal ratio of entanglement coefficients of the new entanglement. And then, for the convenience of different experimenters, an iterative method is explored to approximate the optimal case using adaptable parameters. Moreover, as a schematic experimental implementation, the phonotic

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entanglement transfer will be proposed, by taking use of polarization qudit representation $|j\rangle_d := |(d-1-j)H, jV\rangle$ for $j = 0, \dots, d-1$, where $|H\rangle$ and $|V\rangle$ represent the horizontal and vertical polarizations respectively. The optimal photonic scheme requires the cross-phase modulation technology [44–51], which is based on the weak cross-Kerr nonlinearity [52,53]. Generally, our results may be useful for general quantum information processing based on qudit systems.

The rest of this paper is organized as follows. Section 2 is firstly devoted to finding new entanglement under the majorization order of two known entanglements. Some simulations are also shown that all entanglements in the unit sphere can be deterministically transformed into new entanglements which lie in a small area of the unit sphere. And then an equivalent optimization problem is proposed for the general entanglement transfer from the new entanglement to the goal state. The optimal scheme will be realized iteratively with some adaptable parameters. Section 3 contributes to the optimal schematic photonic entanglement transfer with the help of auxiliary photon and the weak cross-Kerr nonlinearity while the last section concludes this paper.

## 2. General bipartite entanglement transfer

Suppose that two participants Alice and Bob share an entanglement

$$|\Psi\rangle_{AB} = \sum_{j=0}^{d-1} a_j |j\rangle_A |j\rangle_B, \tag{1}$$

where qudit systems $A$ and $B$ belong to Alice and Bob respectively, and all known real coefficients $a_j$s satisfy $\sum_{j=0}^{d-1} a_j^2 = 1$. Here, all complex phases $\exp(i\theta_j)$ of $a_j$s may be reduced from one local rotation $\sum_{j=0}^{d-1} \exp(-i\theta_j)|j\rangle\langle j|$ by Alice or Bob. In the following, they want to obtain another bipartite entanglement

$$|\Phi\rangle_{AB} = \sum_{i=0}^{d-1} b_i |i\rangle_A |i\rangle_B, \tag{2}$$

where new real coefficients $b_j$s satisfy $\sum_{j=0}^{d-1} b_j^2 = 1$. This task may be completed deterministically if two coefficient sets $\{a_0^2, a_1^2, \dots, a_{d-1}^2\}$ and $\{b_0^2, b_1^2, \dots, b_{d-1}^2\}$ satisfy the majorization condition stated in Ref. [20]. However, this is impossible for most entanglements $|\Psi\rangle$ and $|\Phi\rangle$. So, we have to find other methods to address general entanglement transfer. In fact, this problem may be solved by evaluating the optimal success probability and present the optimal probabilistic scheme. Especially, for the case of $b_0 = b_1 = \dots = b_{d-1} = 1/\sqrt{d}$ (the maximal bipartite entanglement $|\Phi\rangle_{AB}$), general entanglement transfer scheme should be reduced to previous entanglement concentration scheme.

### 2.1. Deterministic entanglement transfer

Before completing general entanglement transfer, new bipartite entanglement may be obtained from $|\Psi\rangle_{AB}$ in terms of $|\Phi\rangle_{AB}$ under the majorization order. In fact, assume that

$$a_0^2 \le a_1^2 \le \dots \le a_{d-1}^2, \tag{3}$$

$$b_0^2 \le b_1^2 \le \dots \le b_{d-1}^2. \tag{4}$$

Let

$$c_i^2 = \frac{\sum_{j=0}^{i} a_j^2}{\sum_{j=0}^{i} b_j^2}, \quad i = 0, 1, \dots, d-1. \tag{5}$$

If

$$c_0^2 \le c_1^2 \le \dots \le c_{d-1}^2, \tag{6}$$

i.e., $a_0^2/b_0^2 \le a_1^2/b_1^2 \le \dots \le a_{d-1}^2/b_{d-1}^2$, the entanglement coefficient set $\{b_0^2, b_1^2, \dots, b_{d-1}^2\}$ is named as *majored* by the entanglement coefficient set $\{a_0^2, a_1^2, \dots, a_{d-1}^2\}$ [20]. In this case, $|\Phi\rangle_{AB}$ can be determinately transformed to $|\Psi\rangle_{A,B}$. Meanwhile, $|\Psi\rangle_{A,B}$ cannot be determinately changed into another entanglement $|\Psi'\rangle$ which may be used to improve the success probability of the bipartite entanglement transfer. This is because that there does not exist the entanglement coefficient set $\mathcal{E}$ which is majored by the entanglement coefficient set $\{a_0^2, a_1^2, \dots, a_{d-1}^2\}$, and $\{b_0^2, b_1^2, \dots, b_{d-1}^2\}$ is majored by $\mathcal{E}$. It may be simply stated that they cannot find another entanglement $|\Psi'\rangle$ with coefficient set $\mathcal{E}$ between $\{a_0^2, a_1^2, \dots, a_{d-1}^2\}$ and $\{b_0^2, b_1^2, \dots, b_{d-1}^2\}$ in terms of the majorization order. For other cases that two coefficient sets $\{b_0^2, b_1^2, \dots, b_{d-1}^2\}$ and $\{a_0^2, a_1^2, \dots, a_{d-1}^2\}$ have no direct majorization order, let

$$\alpha_{j_0}^2 = c_{j_0}^2 = \min_j \{c_j^2, j = 0, 1, \dots, d-1\}, \tag{7}$$

$$\alpha_{j_1}^2 = \frac{\sum_{i=j_0+1}^{j_1} a_i^2}{\sum_{i=j_0+1}^{j_1} b_i^2} = \min_k \left\{ \frac{\sum_{i=j_0+1}^{k} a_i^2}{\sum_{i=j_0+1}^{k} b_i^2}, \ k = j_0+1, \dots, d-1 \right\}, \tag{8}$$

$$\vdots \tag{9}$$

$$\alpha_{j_r}^2 = \frac{\sum_{i=j_{r-1}+1}^{d-1} a_i^2}{\sum_{i=j_{r-1}+1}^{d-1} b_i^2} = \min_k \left\{ \frac{\sum_{i=j_{r-1}+1}^{k} a_i^2}{\sum_{i=j_{r-1}+1}^{k} b_i^2}, \ k = j_{r-1}+1, \dots, d-1 \right\}. \tag{10}$$

It easily follows that $0 \le \alpha_{j_0}^2 \le \alpha_{j_1}^2 \le \dots \le \alpha_{j_r}^2 \le 1$, $1 \le j_0 \le j_1 \le \dots \le j_r = d-1$ and $\mathcal{E} = \{\alpha_0^2 b_0^2, \alpha_0^2 b_1^2, \dots, \alpha_0^2 b_{j_0}^2, \ \alpha_1^2 b_{j_0+1}^2, \dots, \alpha_1^2 b_{j_1}^2, \ \dots, \alpha_r^2 b_{j_{r-1}+1}^2, \dots, \alpha_r^2 b_{j_r}^2\}$ is an entanglement coefficient set (one probability distribution). Thus the entanglement coefficient set $\{b_0^2, b_1^2, \dots, b_{d-1}^2\}$ is majored by the following coefficient set $\mathcal{E}$ and $\mathcal{E}$ is majored by $\{a_0^2, a_1^2, \dots, a_{d-1}^2\}$. It means that the entanglement

$$|\Psi'\rangle = \sum_{t=0}^{j_0} \alpha_0 b_t |tt\rangle_{AB} + \sum_{k=1}^{r} \sum_{s=j_{k-1}+1}^{j_k} \alpha_k b_s |ss\rangle_{AB} \tag{11}$$

may be deterministically obtained from $|\Psi\rangle_{AB}$ under the LOCC. Moreover, $\{b_0^2, b_1^2, \dots, b_{d-1}^2\}$ is majored by the coefficients of $|\Psi'\rangle_{AB}$. Some

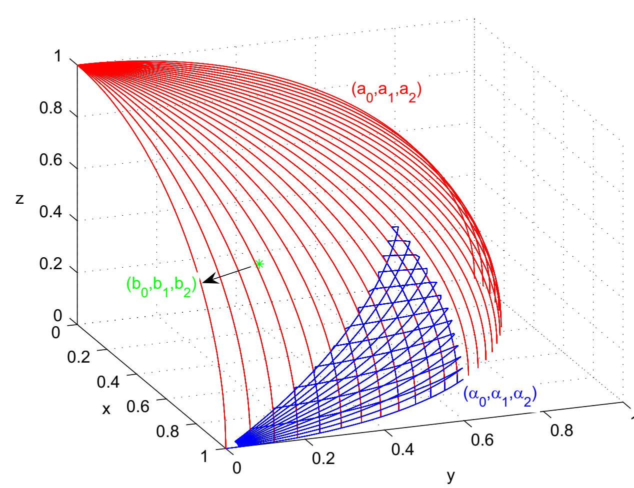


**Fig. 1.** Bipartite entanglement concentration under the LOCC. $|\Psi\rangle_{AB} = a_0|00\rangle + a_1|11\rangle + a_2|22\rangle$ is the initial bipartite entanglement shown as the red figure, $|\Phi\rangle_{AB} = 1/\sqrt{3}(|00\rangle + |11\rangle + |22\rangle)$ is the final bipartite entanglement shown as the green point, and $|\Psi'\rangle_{AB} = \alpha_0|00\rangle + \alpha_1|11\rangle + \alpha_2|22\rangle$ is the new bipartite entanglement defined in Eq. (11) shown as the blue figure. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)