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Torsional vibrations of a column of fine-grained material: A gradient elastic approach

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ABSTRACT

The gradient theory of elasticity with damping is successfully employed to explain the experimentally observed shift in resonance frequencies during forced harmonic torsional vibration tests of columns made of fine-grained material from their theoretically computed values on the basis of the classical theory of elasticity with damping. To this end, the governing equation of torsional vibrations of a column with circular cross-section is derived both by the lattice theory and the continuum gradient elasticity theory with damping, with consideration of micro-stiffness and micro-inertia effects. Both cases of a column with two rotating masses attached at its top and bottom, and of a column fixed at its base carrying a rotating mass at its free top, are considered. The presence of both micro-stiffness and micro-inertia effects helps to explain the observed natural frequency shift to the left or to the right of the classical values depending on the nature of interparticle forces (repulsive or attractive) due to particle charge. A method for using resonance column tests to determine not only the shear modulus but also the micro-stiffness and micro-inertia coefficients of gradient elasticity for fine-grained materials is proposed.

1. Introduction

The most widely used laboratory test for measuring the shear modulus, G, of soils under low-strain conditions is the resonant column test (Kramer, 1996). To this end, the soil column (solid or hollow) is subjected to harmonic torsional vibrations and the strain amplitude is recorded for a series of loading frequencies. The lowest resonance frequency is the first natural frequency of the soil column. This frequency is used to back-calculate, on the basis of the classical wave equation governing the torsional vibrations of the column, the shear wave propagation velocity c. The shear modulus G, is then determined from c and the pre-specified mass density ρ of the soil material.

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Notwithstanding the validity and usefulness of the test, one could question the accuracy of the above procedure for determining G on the basis of only the first (fundamental) natural frequency of the specimen, by arguing that use of higher natural frequencies may lead to different, possibly more accurate, values of G. Furthermore, one could point out that, since the measured resonance frequency is in reality a damped frequency, the analytical frequency equation used should include damping easily measured during the resonant column test. Finally, it has been recently observed by Richter (2006) during resonant column tests involving fine-grained materials that a shift in resonance frequencies to the left or to the right of their theoretically computed values by classical elastodynamic theory occurs. This shift corresponds to repulsive and attractive granular materials, respectively, depending on particle electric charge (Richter, 2006). However, these interesting experimental observations could not be explained by the classical theory of elasticity.

In this paper, an effort is made to explain theoretically the experimentally observed frequency shifting and suggest a way for a more rational computation of G. This is accomplished by introducing into the governing equation of torsional elastic vibrations with damping of a beam of circular cross-section microstructural effects, i.e., both micro-stiffness and micro-inertia with the aid of the lattice theory or the continuum gradient elasticity theory with two microstructural constants by following Polyzos and Fotiadis (2012) and Mindlin (1964), respectively. Both approaches lead to a governing equation of torsional motion including two length scale parameters, in addition to the classical shear modulus G, namely, the micro-stiffness or gradient coefficient g and the micro-inertia coefficient h. More specifically, it is shown that depending on the relation between the magnitudes of g and h, one can predict whether the aforementioned frequency shifting will be to the left or to the right of the classically computed eigen-frequencies. Furthermore, since the system is characterized by three elastic constants instead of just one in the classical case, one can possibly engage the first three experimentally obtained resonance frequencies for computing G, g and h, thereby obtaining a more rational value of G than by classical means.

Generalized elasticity theories encompassing microstructural effects have been successfully employed for studying torsional vibrations of beams modeling nanotubes. In this context, Gheshlaghi et al. (2010) utilized the modified couple stress theory with one length scale parameter, Kahrobaiyan et al. (2011) a strain gradient theory with three length scale parameters and Lim et al. (2012) a nonlocal stress theory with one length scale parameter. However, none of the above works considers micro-inertia effects which, as demonstrated in Georgiadis et al. (2004), Askes et al. (2008), Papargyri-Beskou et al. (2009), Fafalis et al. (2012) and Dontsov et al. (2013), are not only important alongside the micro-stiffness ones, but also characterize the dynamic behavior of a wide class of materials and structures. Further, none of the above investigations considers the effect of internal viscoelastic damping on the response. In this work, both microstructural parameters play an equally important role and help to explain the dynamic behavior of granular beams under torsional vibrations. Besides, the effect of internal viscoelastic damping on the response is considered for a more realistic treatment of the problem. Additional discussion on theoretical aspects of gradient elasticity theory is presented in Section 3.

2. Resonant column test results for fine-grained material

In his doctoral dissertation, Richter (2006) presented experimental results on the dynamic behavior of fine-grained soils under cyclic loading, which find applications in a variety of soil dynamics problems. For this purpose, he employed model materials instead of natural fine-grained soil, i.e., α-Al_2O_3 powder (hard compact particles) and Laponite (synthetic clay) representing silt and clay, respectively. A good part, but not all, of the work in Richter (2006) can also be found in Richter and Huber (2003, 2004).

Fine-grained materials like α-Al_2O_3 have a mean particle diameter $d_{50}=0.8$ μm and exhibit a fabric depending on the surface forces between the grains, which are mainly responsible for the formation of the grain skeleton. In a fabric of attractive particles (particle charge pH=9.1), interparticle friction results in low density, while in a fabric of repulsive particles (particle charge pH=4.0) interparticle repulsion prevents friction and enhances densification, as shown in Fig. 1 taken from Richter (2006). All these materials are, naturally, fine-grained by geotechnical standards.

Richter (2006) reported on experimental results from resonant column tests conducted on fine-grained saturated α-Al_2O_3 columns subjected to torsional harmonic vibrations with the goal of determining the shear modulus G and the damping ratio D of these materials. The tests were conducted for small to medium values of engineering shear strains γ, i.e., for $\gamma=10^{-7}$ to 10^{-3}, for values of frequency f varying from 0 to 5600 Hz and for values of confining pressure p' varying between 20 and 320 kPa.

Fig. 2a represents the resonant column test apparatus used by Richter (2006), while Fig. 2b its mathematical model. The height L and the radius r of the specimen are equal to 0.10 m and 0.05 m, respectively, the polar moment of inertia of the cross-section of the specimen $I_p=\pi r^4/2=98.125\times10^{-3}$ m^4, while the mass moments of inertia of the top and bottom masses of the apparatus are $J_L=0.854549\times10^{-3}$ kg m^2 and $J_0=57.352325\times10^{-3}$ kg m^2, respectively. Figs. 3 and 4 contain representative results from Richter (2006) corresponding to the cases of attractive (pH=9.1) and repulsive (pH=4.0) particles, respectively, for a confining pressure of 20 kPa. Both figures depict the normalized resonance factor R_t/R_b as function of frequency f, where the resonance factors R_t and R_b are defined as the ratios of the amplitudes of vibration A_t and A_b at the top and bottom, respectively, of the soil column to the static torsional angle θ_s. Furthermore, Figs. 3 and 4 also depict