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Long distance cavity entanglement by entanglement swapping using atomic momenta

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ABSTRACT

We propose a simple technique to generate entanglement between distant cavities by using entanglement swapping involving atomic momenta. For the proposed scheme, we have two identical atoms, both initially in their ground state, each incident on far apart cavities with particular initial momenta. The two cavities are prepared initially in superposition of zero and one photon state. First, we interact each atom with a cavity in a dispersive way. The interaction results into atom–field entangled states. Then we perform EPR state measurement on both atomic momentum states which is an analog of Bell measurement. The EPR state measurement is designed by passing the atoms through cavity beam splitters which transfers the atomic momentum state into the superposition state. Finally, these atoms are detected by the detector. After the detection of the atoms, we can distinguish that cavities in one of the Bell states. This process leads to two distant cavity fields entanglement.

1. Introduction

Entanglement, a non-local trait of quantum theory, has many applications in quantum informatics [1]. The cavity quantum electrodynamics (QED) techniques are used to generate atom–field, atom–atom and field–field entanglement [2]. Entanglement in the atomic external degrees of freedom using Bragg diffraction is also proposed [3,4]. Bragg diffraction of atomic de-Broglie waves from optical cavity also covers some aspects of quantum information [5,6].

Entanglement swapping, an important technique of entanglement, entangles two parties that have never interacted before. Entanglement swapping between two photons that have never coexisted is demonstrated [7]. Bell measurements are much useful in quantum communication protocols such as teleportation [8] and entanglement swapping [9]. Entanglement swapping is used in quantum repeaters [8], in order to overcome the limiting effect of photon loss in long-range quantum communication.

In this paper, we use a simple technique i.e. atomic interferometry for swapping entanglement between atoms and cavities. This way we are able to entangle distant cavities without direct interaction. For the proposed scheme, we have two cavities which are in superposition state of zero and one photon. The cavity superposition state is experimentally demonstrated by Rauschenbeutal et al. [10]. First, we interact two atoms, initially in their ground state having momentum $|P_0^i\rangle$, $i\epsilon\{1,2\}$ each with a cavity in the Bragg diffraction regime. Bragg scattering allows only one of the two directions of propagation for each atom along the cavity field which are the incident and exactly opposite one. The detuning is large as compared to single photon Rabi frequency and hence atom practically stays in the ground state and the state of the field does not change. Here we take first-order Bragg diffraction for simplicity, however higher order Bragg scattering can be taken into account in the same fashion in order to allow larger separation between the atoms after the interaction. The non-resonant interaction entangles the atoms in their external degrees of freedom i.e. in their momentum states with the cavities. Then these entangled atoms are passed through beam splitter. For this purpose we use two beam splitters, one for non-deflected atomic momentum state and the other for deflected atomic momentum state. The beam splitter brings the atomic momentum state of these indistinguishable atoms in the superposition state. A cavity in the superposition state of zero and one photon can be used as a beam splitter [11]. At last, after passing through the beam splitter, these identical atoms are detected. Here, we use four detectors for four possible momentum splits. The detection process

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gives us the information that the two cavities are in the Bell state. Thus entanglement between atoms is swapped to that between two far away cavities.

Our paper proceeds as follow: in Section 2, we explain the Bragg diffraction of atom from the cavity field and the formation of atom–field entanglement. In Section 3, we analyze the action of beam splitter which transfers the atomic momentum component into the superposition state. We then briefly explain the detection process and the final result. Finally we conclude in Section 4 and give experimental parameters to perform our proposed scheme in the laboratory.

2. Bragg atom–field interaction

For the proposed scheme, we first entangle two atoms with their respective cavity fields by atom–field interaction in the Bragg regime. For the purpose, we consider two atoms, A_1 and A_2, both initially in their ground state, g_1 and g_2, having transverse momentum state, $|P^i_{l_0}\rangle$, where $i=1$, 2 stands for atoms A_1 and A_2 and $P_{l_0}=(l_0/2)\hbar k$ with l_0 being a positive even integer. We have two cavities, C_1 and C_2, which are in the superposition state of zero and one photon i.e. $(|0\rangle+|1\rangle)/\sqrt{2}$ [3] as shown in Fig. 1. This superposition can be generated by first passing a two level atom in its excited state for half a Rabi cycle through the field. We dispersively interact atom, A_1, with cavity, C_1, and atom, A_2, with cavity, C_2. The off-resonant interaction is followed to avoid decoherence that stems from spontaneous emission. Large detuning and large interaction time ensure conservation of energy which leads to only two possible directions of scattering for atoms, first the incident one, P_{l_0}, and second exactly opposite to the incident transverse momentum direction, P_{-l_0}. The off-resonant Bragg diffraction invokes only the virtual transition among different atomic levels [11]. The initial state vector for the system before interaction is

$$\left|\Psi(0)\right\rangle=\frac{1}{2}\sum_{i=1,2}\left(\left|0_i\right\rangle+\left|1_i\right\rangle\right)\otimes\left|g_i,P^{(i)}_{l_0}\right\rangle. \tag{1}$$

Total Hamiltonian governing this atom–field interaction under the dipole and rotating wave approximation with atom of mass, M, and centre of mass momentum, P, is [3]

$$\hat{H}=\frac{\hat{P}^2}{2M}+\frac{1}{2}\hbar\omega_0\hat{\sigma}_z+\hbar\nu\hat{a}^\dagger\hat{a}+\hbar g\cos(k\hat{x})[\hat{\sigma}_+\hat{a}+\hat{\sigma}_-\hat{a}^\dagger]. \tag{2}$$

Here, $\hat{\sigma}_\pm$ and $\hat{\sigma}_Z$ are the Pauli operators, $\hat{x}$ is the position operator of atom, $\hat{a}$ $(\hat{a}^\dagger)$ is the field annihilation (creation) operator, g is the vacuum Rabi frequency and Δ is the detuning between the atomic transition frequency, ω_0, and the field frequency, ν. We follow the large detuning case where we have no direct atomic transition and it is rare to find the atom in their excited state. Hence, the system may be governed by following effective Hamiltonian, under the adiabatic approximation as

$$\hat{H}_{\text{eff}}=\frac{\hat{P}^2}{2M}-\frac{\hbar|g|^2}{2\Delta}\hat{n}\hat{\sigma}_-\hat{\sigma}_+(\cos 2k\hat{x}+1). \tag{3}$$

The state of each ith atom–field pair at any time t is given as

$$\left|\Psi(t)\right\rangle=\frac{1}{\sqrt{2}}\sum_{l=-m}^{m}\left(C_{0,\tilde{P}_l}\left|0,g_i,\tilde{P}^{(i)}_l\right\rangle+C_{1,\tilde{P}_l}\left|1,g_i,\tilde{P}^{(i)}_l\right\rangle\right), \tag{4}$$

where m is the total number of the orders of deflections and $\tilde{P}_l=P_{l_0}+l\hbar k$, l being an even integer. Time evolution of the state vector is given by the Schrodinger equation

$$i\hbar\frac{\partial|\Psi(t)\rangle}{\partial t}=H_{\text{eff}}|\Psi(t)\rangle \tag{5}$$

We have

$$\cos 2k\hat{x}|\tilde{P}_l\rangle\backsim|\tilde{P}_{(l+2)}\rangle+|\tilde{P}_{(l-2)}\rangle \tag{6}$$

and we drop the unchanged atomic ground state vector $|g_i\rangle$. Under condition of Bragg scattering with only two possible directions of deflection $l=0$ with $\tilde{P}_0=P_{l_0}$ and $l=-l_0$ with $\tilde{P}_{-l_0}=P_{-l_0}$. Thus we obtain the state of each ith atom–field pair after interaction as [3,12]

$$\left|\Psi(t)\right\rangle=\frac{1}{\sqrt{2}}\left(|0_i,P^{(i)}_{l_0}\rangle+C_{1,l_0}(t)|1_i,P^{(i)}_{l_0}\rangle+C_{1,-l_0}(t)|1_i,P^{(i)}_{-l_0}\rangle\right) \tag{7}$$

where $C_{n,\pm l_0}$ is the probability amplitude of the atom exiting with momentum P_{+l_0} or P_{-l_0} when there are n photons in the field and is given as

$$C_{n,\pm l_0}(t)=e^{-iA_nt}\left[C_{n,\pm l_0}(0)\cos\left(\frac{1}{2}B_nt\right)+iC_{n,\mp l_0}(0)\sin\left(\frac{1}{2}B_nt\right)\right] \tag{8}$$

where

$$A_n\equiv\begin{cases}-\dfrac{(|g|^2n/4\Delta)^2}{\omega_{\text{rec}}(l_0-2)(2)} & \text{for } l_0\neq 2\\ 0 & \text{for } l_0=2\end{cases}$$

and

$$\left|B_n\right|\equiv\begin{cases}\dfrac{(|g|^2n/2\Delta)^{l_0/2}}{(2\omega_{\text{rec}})^{l_0/2-1}[(l_0-2)(l_0-4)\ldots 4.2]} & \text{for } l_0\neq 2\\ |g|^2n/2\Delta & \text{for } l_0=2\end{cases}$$

Initially both atoms are sent with momentum P_{l_0}, so probability of finding the exiting atom in either directions flips as a cosine function of interaction time. We adjust the interaction time of atoms with fields to ensure that if there is one photon in the fields, the atoms definitely get deflected. The adjusted time is thus $t=r\pi/|B_n|$, where r is an odd integer. For first-order Bragg scattering, this time simplifies to $t=2r\pi\Delta/|g|^2$. The wave function of the two atom–field pairs is

$$\left|\Psi(t)\right\rangle=\left[\frac{1}{\sqrt{2}}\left(|0_1,P^{(1)}_{l_0}\rangle+ie^{-i\phi}|1_1,P^{(1)}_{-l_0}\rangle\right)\right]\otimes\left[\frac{1}{\sqrt{2}}\left(|0_2,P^{(2)}_{l_0}\rangle+ie^{-i\phi}|1_2,P^{(2)}_{-l_0}\rangle\right)\right], \tag{9}$$

where $\phi=r\pi A_1/B_1$. The atoms in their external degrees of freedom become entangled with their respective cavity fields. The combined state of the system can be written as

$$\left|\Psi(t)\right\rangle=\tfrac{1}{2}\left(\left|0_1,0_2,P^{(1)}_{l_0},P^{(2)}_{l_0}\right\rangle+ie^{-i\phi}\left|0_1,1_2,P^{(1)}_{l_0},P^{(2)}_{-l_0}\right\rangle+ie^{-i\phi}|1_1,0_2,P^{(1)}_{-l_0},P^{(2)}_{l_0}\rangle-e^{-i2\phi}|1_1,1_2,P^{(1)}_{-l_0},P^{(2)}_{-l_0}\rangle\right). \tag{10}$$

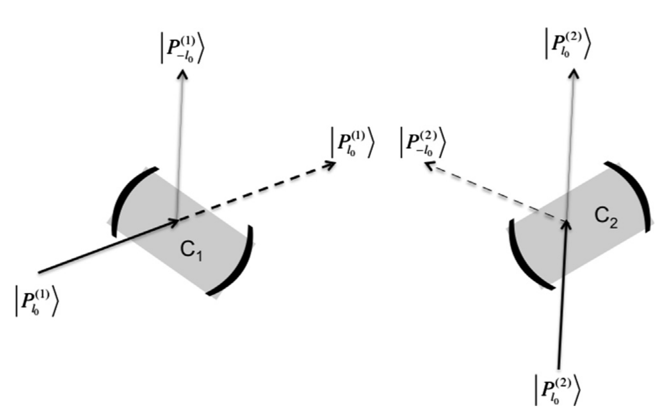

Fig. 1. We show dispersive interaction of atoms with cavity fields. The atoms with initial momentum, $|P^i_{l_0}\rangle$, interact with the cavities which are in superposition of zero and one photon state. The interaction time is set such that when the cavities are in zero photon state, the atoms do not get deflected and have same momentum $|P^i_{l_0}\rangle$ as initial one. For one photon state of the cavities, the atoms are deflected and have momentum $|P^i_{-l_0}\rangle$.