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Magnetic field-induced martensitic phase transformation in magnetic shape memory alloys: Modeling and experiments

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ABSTRACT

In this work, a continuum based model of the magnetic field induced phase transformation (FIPT) for magnetic shape memory alloys (MSMA) is developed. Hysteretic material behaviors are considered through the introduction of internal state variables. A Gibbs free energy is proposed using group invariant theory and the coupled constitutive equations are derived in a thermodynamically consistent way. An experimental procedure of FIPT in NiMnCoIn MSMA single crystals, which can operate under high blocking stress, is described. The model is then reduced to a 1-D form and the material parameter identification from the experimental results is discussed. Model predictions of magneto-thermo-mechanical loading conditions are presented and compared to experiments.

1. Introduction

Magnetic shape memory alloys (MSMAs) are best known for their unique ability to produce magnetic field induced strains (MFISs) up to 10% under a magnetic field (O'Handley, 1998; O'Handley et al., 2000; Müllner et al., 2003; Shield, 2003). Some of the commonly used MSMA material systems are NiMnGa (Murray et al., 2001a; O'Handley et al., 2003; Heczko et al., 2003; Likhachev et al., 2004), FePd (James and Wuttig, 1998; Yamamoto et al., 2004) and NiMnX, where X = In, Sn, Sb (Sutoua et al., 2004). The unique magneto-mechanical coupling makes MSMAs promising materials for multifunctional structures, actuator and sensor applications (Pasquale, 2003; Tellinen et al., 2002; Sarawate and Dapino, 2006, 2007; Karaman et al., 2007).

The coupled MSMA behaviors can be modeled by considering the material as an electromagnetic continuum. Extensive work on different electromagnetic formulations had been proposed in the literature (Toupin, 1956, 1960; Penfield and Haus, 1967; Hutter et al., 2006; Eringen and Maugin, 1990) based on different notions of breaking up *long range* and *short range* forces. In a recent work, DeSimone (1993) and DeSimone and Podio-Guidugli (1996) proposed a continuum theory for deformable ferromagnetic material. Dorfmann and Ogden (2004, 2005) derived a theory of nonlinear magneto-elasticity for magneto-sensitive elastomers. McMeeking and Landis (2005) and McMeeking et al. (2007) presented a study of electrostatic forces on large deformations of polarizable material. A theory for the equilibrium response of magnetoelastic membranes is formulated by Steigmann (2004, 2009). Guidugli et al. (2010) formulated a continuum theory for the evolution of magnetization and temperature in a rigid magnetic body for ferro-/paramagnetic transition. The variational formulations for

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general magneto-mechanical materials have been proposed by many authors (Kankanala and Triantafyllidis, 2004; Ericksen, 2006; Bustamante et al., 2008; Miehe et al., 2011a, 2011b).

The macroscopically observable MFIS in MSMAs is caused by the microstructural *reorientation of martensitic variants* (O'Handley et al., 2000; Karaca et al., 2006), *field induced phase transformation* (Sutoua et al., 2004; Kainuma et al., 2006; Karaca et al., 2007, 2009) or a combination of the two mechanisms. In the variant reorientation mechanism, the variants have different preferred directions of magnetization and the magnetic field is applied to select certain variants among others, which results in the macroscopic shape change.

A major drawback for the field-induced variant reorientation mechanism is the comparatively low blocking stress. With this mechanism, when the applied stress level is more than 10 MPa (Ganor et al., 2008), the MAE is insufficient to overcome the energy required for twin boundary motion. Thus, magnetic field favored martensitic variant does not grow and field induced macroscopic shape change is not observed. The limited availability of MAE restricts the variant reorientation mechanism to work above certain stress levels.

The limitation of the available magnetic energy and thus low blocking stress in field-induced variant reorientation can be overcome by the magnetic field induced martensitic phase transformation. This mechanism is analogous to the temperature induced martensitic transformation in conventional shape memory alloys (SMAs). The *Zeeman energy* (ZE),[1] which depends on the difference between the saturation magnetizations of the austenitic and martensitic phases, is converted to mechanical energy through the magnetic field induced phase transformation. In the NiMnCoIn material system, the Zeeman energy can be substantial as the austenitic phase is ferromagnetic while the martensitic phase is antiferromagnetic. The available magnetic energy (ZE) involved in field-induced phase transformation (FIPT) is much higher than the magnetic energy (MAE) associated with the variant reorientation mechanism. Karaca et al. (2009) showed that the ZE in $Ni_{45}Mn_{36.5}Co_5In_{13.4}$ is one order of magnitude higher than the MAE of NiMnGa alloys. This unique characteristic of highly available magnetic energy in the FIPT can lead to large MFIS and high actuation stresses. Moreover, unlike MAE, ZE is independent of crystallographic orientation and provides an opportunity to utilize polycrystals for actuator application (Karaca et al., 2007). Another unique property of this material is its ability to function like conventional SMAs.

The NiMnGa system, which is widely used for variant reorientation, also exhibits FIPT under stress levels of approximately 20 MPa with 0.5% MFIS (Karaca et al., 2007). For the NiMnCoIn system, Kainuma et al. (2006) found that a 4 T magnetic field can recover 3% pre-applied strain in martensite at room temperature. Wang et al. (2008) also reported reversible FIPT under 50 MPa with the application of a 5 T magnetic field with unknown MFIS values using in situ high energy XRD measurement. In the present work, the effect of simultaneous application of high magnetic field (16 T) and high stress (110 MPa) on the transformation is investigated.

There are two major modeling approaches for variant reorientation mechanism. In microstructural based models, the resulting macroscopic strain and magnetization response are predicted by minimizing a free energy functional. This functional includes terms related to the magnetostatic energy, the elastic energy and the *Magnetic Anisotropy Energy* (MAE)[2] within the martensitic twin variant. Details on the microstructural based modeling approach can be found in James and Wuttig (1998), DeSimone and James (1997, 2002, 2003), O'Handley (1998), and Murray et al. (2001b). The second approach to study the material behavior is through thermodynamics based phenomenological modeling (Hirsinger and Lexcellent, 2003a, 2003b; Kiefer and Lagoudas, 2005, 2009; Kiefer et al., 2006). Most recent development of a variational modeling of variant reorientation in MSMAs can be found in Wang and Steinmann (2012). A magnetostatic stability analysis for Ni_2MnGa material system is reported in Haldar et al. (2010). The effects of magnetic body force and couple on the variant reorientation mechanism are investigated through a coupled boundary value problem in Haldar et al. (2011). A numerical technique to correct the constitutive responses from the demagnetization effect is also described in Haldar et al. (2011).

Motivated by FIPT experiments, we develop a phenomenological model to capture the magneto-thermo-mechanical material responses of a dissipative system with hysteretic behaviors. Nonlinear kinematics is introduced to capture magneto-mechanical cross-coupling. The model takes into account the loading history dependence and the evolution of internal state variables based on a maximum dissipation principle. Moreover, single crystal anisotropy, which was not considered in the previous MSMA modeling, is introduced for the model development. To the best of the authors' knowledge, phenomenological modeling of FIPT is not reported so far in the literature.

The structure of the paper is as follows: in Section 2, we develop a thermodynamic based continuum framework and we identify the external and internal state variables. A specific form of the Gibbs free energy is proposed in Section 3 and the constitutive equations are derived from the proposed Gibbs free energy. The model is then reduced to 1-D in Section 4. A detailed experimental procedure is presented in Section 5 and the model is calibrated from the experiments in Section 6. Finally, we present model predictions in Section 7.

2. Continuum description of the model

We aim to propose a phenomenological model for FIPT from the experimental observations. Magneto-mechanical experimental conditions are schematically presented in Fig. 1(a). In the experiments, a magnetic field $\mathbf{H}_a$ is applied through a

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[1] Energy of a magnetized body in an external applied magnetic field.

[2] MAE is the energy required to rotate the magnetization vector from the direction of the easy axis to the direction of the hard axis.