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Instability of a magnetoelastic layer resting on a non-magnetic substrate

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ABSTRACT

Magnetorheological elastomers (MREs) are ferromagnetic particle impregnated rubbers whose mechanical properties are altered by the application of external magnetic fields. Due to their coupled magneto-mechanical response, MREs are finding an increasing number of engineering applications. One such application is in haptics, where the goal is to actively control surface roughness. One way to achieve this is by exploiting the unstable regime of MRE substrate/layer assemblies subjected to transverse magnetic fields. In this work, we study the response of such an assembly subjected to a transverse magnetic field and in-plane stress. The layer is made up of a transversely isotropic MRE material, whose energy density has been obtained experimentally, while the substrate is a non-magnetic isotropic pure polymer/gel. An analytical solution to this problem based on a general, finite strain, 2D continuum modeling for both the MRE layer and the substrate shows that for adequately soft substrates there is a finite-wavelength buckling mode under a transverse magnetic field. Moreover, the critical magnetic field can be substantially reduced in the presence of a compressive stress of the assembly, thus opening the possibility for haptic applications operating under low magnetic fields.

1. Introduction

Magnetorheological elastomers (MREs) are ferromagnetic particle impregnated rubbers that can be deformed by external magnetic fields (e.g. see Rigbi and Jilkén, 1983; Ginder et al., 1999). These composites are members of a wide class of materials that exhibit multi-physics couplings and termed "*active materials*". In virtue of the strong interactions between their magnetic and elastic response, MRE's are considered for a wide number of engineering applications, such as sensors, actuators, vibrations dampers or haptic devices. It is this latter category of applications that motivates the present study and more specifically the possibility of inducing deformation patterns in MRE layers through external magnetic fields and controlled by mechanical loads.

More specifically, of interest here is a MRE layer bonded on a non-magnetic substrate and subjected to a combination of magnetic and mechanical loads: transverse (along the thickness direction) magnetic field and axial (parallel to the interface)

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compression. Either loading by itself can lead to bifurcation (buckling) of the MRE layer. For the purely magnetic loading case, Moon and Pao (1968) studied the problem of magnetoelastic buckling of a thin metallic ferromagnetic plate subjected to a transverse magnetic field using structural models. In that work, the plate is initially flat and stable. When the magnetic field reaches a critical value, the plate rotates and the corresponding buckling wavelength is comparable to the finite length of the thin plate. The instability of the plate subjected to a transversely magnetic field can be interpreted as the well-known compass effect in magnetism, where an unconstrained rod/plate tends to align with the applied magnetic field. In the present work, the presence of the non-magnetic substrate penalizes the energy of the long wavelength modes in the unbounded MRE layer/substrate system, leading to finite wavelengths for the critical instability mode (i.e., mode corresponding to the lowest value of the applied external magnetic field).

In a subsequent study, Pao and Yeh (1973) revisited this problem using a continuum theory of magnetoelasticity which, upon linearization, yields the buckling results of Moon and Pao (1968). In addition to these theoretical investigations, one should also mention associated experimental studies (e.g. Wallerstein and Peach, 1972; Popelar, 1972; Miya et al., 1978). The bifurcation problem of an MRE block under plane strain conditions and subjected to a transverse magnetic field has recently been investigated by Kankanala and Triantafyllidis (2008) using the coupled magneto-mechanical variational formulation proposed by Kankanala and Triantafyllidis (2004). The structural model results of Moon and Pao (1968) have been recovered from the 2D analysis of Kankanala and Triantafyllidis (2008) in the asymptotic limit of a vanishing block aspect ratio.

The problem of purely mechanical loading for the substrate/layer assembly has attracted considerably more attention. There, the instability is due to the axial compression of the subspace, a phenomenon initially pointed out by Biot (1965). The addition of a stiffer thin film bonded on a softer substrate has been the object of numerous subsequent studies (the interested reader is referred to Shield et al., 1994; Chen and Hutchinson, 2004; Huang et al., 2005; Audoly and Boudaoud, 2008a and references quoted therein). The combination of mechanical and magnetic loading to study the stability of composites has appeared very recently with the work of Rudykh and Bertoldi (2013), who have investigated infinite, magnetoactive layered composites. In the present work, and keeping in mind our interest in haptic applications, the goal is to choose the constitutive properties of the MRE layer and the substrate in combination with appropriately applied magnetic and mechanical loading conditions in order to achieve buckling at rather low magnetic fields.

The model studied here consists of an infinite MRE layer perfectly bonded to a non-magnetic substrate and subjected to a transverse magnetic field and lateral in-plane compression. The layer is made up of a transversely isotropic MRE, while the substrate is a non-magnetic isotropic elastomer. A variational, energy-based formulation for conservative problems is used and an analytical solution of the associated 2D (plane strain) problem is presented. More specifically, in Section 2, we present the governing equations of the problem, based on the general variational framework of Kankanala and Triantafyllidis (2004). In this same section, we define the geometry of the problem, select the material properties following the experimental results of Danas et al. (2012) and present the principal solution of the MRE substrate/layer assembly. In Section 3, we describe the bifurcation analysis of the assembly for arbitrary constitutive laws. In Section 4, we present results of the critical applied magnetic field $\widehat{h}_c$ and critical stretch ratio of the interface λ_1^c (a measure of the applied compressive stress) as a function of the MRE substrate/layer shear moduli ratio G_s/G_l. In the case of purely magnetic loading ($\lambda_1=1$), because of magnetic saturation the critical magnetic field is found to increase rapidly with G_s/G_l leading to $\widehat{h}_c$ values that are unrealistic for applications. For the same reason, a bifurcation due to a purely magnetic loading becomes impossible except for very soft substrates. By contrast, a combination of mechanical (compressing the MRE substrate/layer assembly with $\lambda_1<1$ near its purely mechanical instability threshold) and magnetic loads brings down substantially the critical magnetic field and the assembly can bifurcate even for substrates that are stiffer than the layer. The MRE layer anisotropy is found to play an important role in the resulting critical magnetic field. Concluding remarks and suggestions for future work are presented in Section 5. For efficiency, the lengthy intermediate steps in the analysis are given in Appendix, whose last section provides a simple, physically motivated structural model that helps the reader understand why a thin, stiff layer bonded on a soft substrate buckles under a transverse magnetic field and establishes key asymptotic results.

2. Problem formulation

This section presents the equations governing the layer–substrate system under magneto-mechanical loading conditions. More specifically, we first present the variational formulation of the problem and the resulting governing equations (the point-wise Euler–Lagrange equations plus the boundary/interface conditions). We then define the layer–substrate geometry and describe the applied loading. Subsequently, we present the energy density function for the MRE-layer and the non-magnetic substrate. The section concludes with the derivation of the principal solution.

2.1. Governing equations

Following the work of Kankanala and Triantafyllidis (2004, 2008), we assume that the potential energy $\mathcal{P}$ of the system under consideration can be written as (see Eq. (3.1) in Kankanala and Triantafyllidis, 2004 for a detailed proof)

$$\mathcal{P}=\int_V \rho_0\left[\psi(\mathbf{F},\mathbf{M},\mathbf{N})-\mu_0\,\mathbf{M}\cdot\widehat{\mathbf{h}}-\mathbf{f}\cdot\mathbf{u}\right]\mathrm{d}V-\int_{\partial V}\mathbf{T}\cdot\mathbf{u}\,\mathrm{d}S+\int_{\mathbb{R}^3}\frac{\mu_0 J}{2}\mathbf{h}\cdot\mathbf{h}\,\mathrm{d}V. \tag{2.1}$$