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Bifurcation into a localized mode from non-uniform periodic deformations around a periodic pattern of voids

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Bifurcation into a shear band is studied for a porous ductile material subject to a combination of shear loading and tensile or compressive loading in different directions. The material with a periodic array of voids is studied by numerical solutions for a plane strain unit cell model with fully periodic boundary conditions. The fundamental pre-bifurcation solution has been studied before, with focus on ductile fracture under conditions of low stress triaxiality. In the previous studies it has been shown that voids in shear are flattened out to micro-cracks, which rotate and elongate until interaction with neighboring micro-cracks gives coalescence. These failure mechanisms are included in the present study, but here the focus is on the possibility that failure may occur earlier, if bifurcation leads to a shear band crossing over many cells, where the plastic strains inside the band will grow very large, while the overall strains in the material will not increase any further. The unit cell analysis with full periodicity is used both inside and outside the band to find the average behavior in the two material regions. This does not allow for point-wise satisfaction of compatibility and equilibrium along the interface between the two regions, but these conditions can be satisfied on the average. The bifurcation analysis includes determination of the direction along which a shear band is first critical.

1. Introduction

Bifurcations in elastic–plastic solids have been studied by many authors based on the general theory of Hill (1958, 1961). Thus, for the plane strain tensile test both diffuse and localized bifurcation modes have been analyzed by Hill and Hutchinson (1975), and similar studies for axisymmetric conditions are given in Needleman and Tvergaard (1977). For a standard strain hardening elastic–plastic solid with a smooth yield surface and normality of plastic flow these studies show that loss of ellipticity of the governing equations, and thus bifurcation into a shear band, will require unrealistically high stresses of the order of elastic moduli. However, materials with a vertex on the yield surface or with dilatant plastic flow can show bifurcation into a shear band at realistic levels of stress and strain (Rice, 1976; Yamamoto, 1978; Tvergaard et al., 1981; Hutchinson and Tvergaard, 1981). The presence of micro-voids in a standard elastic–plastic solid results in plastic volume change on the macro-level, and therefore localization of plastic flow in shear bands is an important failure mode in porous ductile solids.

When a ductile material deforms under relatively high hydrostatic tension micro-voids contained in the material will tend to grow large and ductile failure will occur by coalescence of neighboring voids (see reviews by Garrison and Moody (1987), Tvergaard (1990), and Benzerga and Leblond (2010)). Another mode of failure has been found in recent studies of ductile porous materials subject to shearing, with little or no superposed hydrostatic tension (Tvergaard, 2008, 2009, 2012; Dahl et al., 2012). Here, the voids flatten out to micro-cracks, which rotate and elongate until interaction with neighboring

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micro-cracks can give coalescence. Thus, under high stress triaxiality the void volume fraction increases until ductile fracture occurs, whereas the void volume fraction disappears under low stress triaxiality, as the voids become micro-cracks. In analyses of cases where micro-cracks form it is important to account for the contact between crack surfaces.

For a hardening porous ductile material described by the model of Gurson (1977) the prediction of bifurcation into a shear band was shown by Yamamoto (1978). Such bifurcations were also predicted by Tvergaard (1981), based on a unit cell model analysis for a material containing a periodic pattern of discretely represented voids. Both the material inside the band and that outside the band were seen as homogenized, and the conditions of compatibility and equilibrium along the band interface between the two types of periodically deformed regions could only be satisfied on average. Good agreement was found between the shear band predictions based on the unit cell model and those based on the Gurson model.

Several experimental investigations have been carried out recently that consider ductile fracture in shear under stress triaxiality near zero. Thus, Bao and Wierzbicki (2004), Beese et al. (2010) and Dunand and Mohr (2011) have used special butterfly specimens to study the effect of stress triaxiality and of the Lode angle in stress states dominated by shear. Haltom et al. (2013) have used a tubular specimen in tension–torsion while Ghahremaninezhad and Ravi-Chandar (2013) have used a modified Arcan test to study the same Al alloy. In both cases scanning electron microscopy was used for detailed observation of the final failure mechanisms. It was found that for this material strongly localized plastic strains develop before any visible damage, and the cavities leading to dimples on the fracture surface must have formed at very large strains just prior to rupture.

In the present paper bifurcation into a shear band is analyzed for ductile materials with a periodic pattern of voids, subject to shear loading. The basic periodic deformation pattern is determined by numerical analyses for a unit cell model with fully periodic boundary conditions, as described by Tvergaard (2012). Also inside the shear band a fully periodic deformation pattern is assumed and the interface conditions between the two fields are satisfied on average. In low triaxiality situations like simple shear the collapse of voids to micro-cracks is accounted for. But the analyses also apply to higher stress triaxialities, where the voids will not collapse, including cases with no shear at all as considered by Tvergaard (1981).

2. Problem formulation

The conditions for bifurcation into a shear band mode to be given here apply to a solid with periodic deformations, such that a deformed unit cell with full periodicity describes the stress and strain states throughout the solid. Thus, it is assumed that the full solid can be built of unit cells that are all identical to the one considered in the analysis. Each unit cell could contain several voids or inclusions. Also, the unit cells could be three dimensional with full 3D periodicity satisfied, or they could be simply planar unit cells with in-plane periodicity. In the present paper the bifurcation procedure will be applied to planar unit cells such as those recently studied by Tvergaard (2012), with one void inside each unit cell.

2.1. The basic periodic deformation pattern.

The material to be considered here has a periodic array of voids, with the initial spacing $2A_0$ in the x^1-direction and the initial spacing $2B_0$ in the x^2-direction. The voids are initially circular cylindrical with radius R_0 and plane strain conditions are assumed. The material is analyzed by considering the unit cell in Fig. 1, containing only one void. Finite strains are accounted for and the analyses are based on a convected coordinate Lagrangian formulation of the field equations, with a Cartesian x^i coordinate system used as reference and with the displacement components on reference base vectors denoted

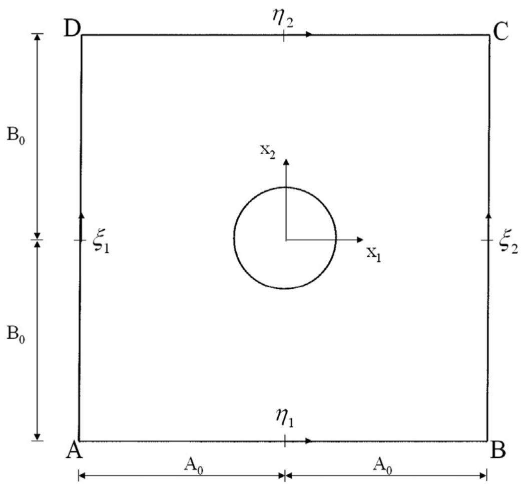

Fig. 1. Coordinates and dimensions for the unit cell analyzed.