Optics Communications 333 (2014) 253–260

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Hermite-Gaussian Vector soliton in strong nonlocal media

Qing Wang [a,b], JingZhen Li [a,*]

[a] Shenzhen Key Laboratory of Micro-Nano Photonic Information Technology, College of Electronic Science and Technology, Shenzhen University, Guangdong, 518060, China
[b] College of Optoelectronic Engineering, Shenzhen University, Guangdong, 518060, China

ARTICLE INFO

Article history:
Received 12 June 2014
Received in revised form
26 July 2014
Accepted 28 July 2014
Available online 11 August 2014

Keywords:
Nonlinear optics
Nonlocal media
Vector soliton
Hermite-Gaussian beam

ABSTRACT

The propagation of two mutually incoherent Hermite-Gaussian (HG) beams in strong nonlocal media was studied. We obtained the evolution equations for the parameters of the two beams and found the condition of forming a HG Vector soliton by variational approach. The numerical result, which accords with the analytical solution very well, shows that a series of vector solitons which consisted of different-order HG beam pairs can be formed in strong nonlocal media. In addition, we found that the phase shifts are not only related to the total incident power, but also related to the orders of the two HG beams.

1. Introduction

Spatial Optical soliton, which is a self-trapped optical beam, exists in that the diffraction is exactly balanced by nonlinearity. Early in 1997, Snyder and Mitchell simplified the nonlocal nonlinear Schrodinger equation (NNLSE) to a simple linear model, namely the Snyder–Mitchell model, and found the accessible soliton [1]. This work stimulates people with a strong interest to study the propagation of optical beam in nonlocal media. So far, a great variety of nonlocal solitons have been studied and a series of achievements have been gained. For instance, D. M. Deng obtained the exact analytical nonlocal Laguerre–Gaussian solutions in a cylindrical coordinate system [2]. The propagation of four-petal Gaussian beams in strongly nonlocal media has been discussed by Z. J. Yang [3]. Families of fundamental and DM solitons are discovered in two-dimensional media with anisotropic semilocal nonlinearity [4]. A class of spiraling elliptic solitons in nonlocal nonlinear media without both linear and nonlinear anisotropy was analyzed by G. Liang [5]. The instability suppression of vector-necklace-ring soliton clusters in different degree of nonlocal media was investigated by M. Shen [6]. Z. Y. Bai demonstrated the dynamics of elegant Ince–Gaussian beams in quadratic-index medium [7] and strongly nonlocal nonlinear media [8]. J. C. Liang found that the Bessel–Gaussian beams only have breather state in strong nonlocal media [9]. The analytical solution of nonlocal Hermite–Gaussian breathers and solitons based on the Snyder–Mitchell model has been investigated [10]. Furthermore, S. W. Zhang found the HG solitons in strong nonlocal media with rectangular boundaries [11]. Buccoliero studied the Laguerre and HG soliton clusters in nonlocal media [12]. Hutsebaut demonstrated that the single-component multihump spatial solitons can travel stably in NLC [13]. The experimental observation of scalar multipole solitons was presented [14] and the stability of multipole-mode solitons in nonlocal nonlinear media was addressed [15]. However, to the best of our knowledge, the HG vector soliton, comprising two incoherent orthogonally polarized beams [16,17], has remained unexplored. In addition, as is widely known, multimodal structure has potential applications value in all-optical control technology [18]. Hence it is necessary to derive the exact analytical solution of HG vector soliton and compare it by direct numerical simulation.

2. Theoretical model and variational approach

The propagation of two incoherent orthogonally polarized HG beams in nonlocal nonlinear media can be described by the coupled nonlocal nonlinear Schrodinger equations (NNLSE) [7,17,19–21]:

$$i\frac{\partial \psi_j}{\partial z}+\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_j +\rho\psi_j\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}R(x-x',y-y')[|\psi_j(x',y',z)|^2 +|\psi_{3-j}(x',y',z)|^2]dx'dy'=0 \quad (1)$$

* Corresponding author.
E-mail address: lijz@szu.edu.cn (J. Li).

http://dx.doi.org/10.1016/j.optcom.2014.07.079
0030-4018/

where ψ_j $(j=1, 2)$ are the paraxial optical beams, $\mu=1/2k$, $\rho=k\eta$, k is the wave number in the media without nonlinearity, η represents the material constant.

The Lagrange density equation, which corresponds to Eq. (1), is given as follows:

$$L=\sum_{j=1,2}\frac{i}{2}(\psi_j^*\frac{\partial\psi_j}{\partial z}-\psi_j\frac{\partial\psi_j^*}{\partial z})-\mu(\left|\frac{\partial\psi_j}{\partial x}\right|^2+\left|\frac{\partial\psi_j}{\partial y}\right|^2)$$
$$+\frac{1}{2}\rho\left|\psi_j\right|^2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}R(x-x',y-y')[\left|\psi_j(x',y',z)\right|^2$$
$$+|\psi_{3-j}(x',y',z)|^2]dx'dy \quad (2)$$

Here we look for the trial solution to Eq. (1) in HG-shaped

$$\psi_j(x,y,z)=A_j(z)H_{n_j}[\frac{x}{a_j(z)}]H_{m_j}[\frac{y}{a_j(z)}]\exp\left[i\theta_j(z)+ic_j(z)(x^2+y^2)-\frac{x^2+y^2}{2a_j^2(z)}\right] \quad (3)$$

where $A_j(z)$ $(j=1, 2)$ are the amplitudes, $\theta_j(z)$ represent the phases of complex amplitude, $a_j(z)$ are the widths and $c_j(z)$ are the phase-front curvatures of the two beams.

The characteristic length of the strong nonlocal media is larger than the beam width, therefore the response function can be expanded twice and reduced as follow [19,20]

$$R(x-x',y-y')\approx R_0-\frac{1}{2}\gamma_x(x-x')^2-\frac{1}{2}\gamma_y(y-y')^2 \quad (4)$$

where $R_0=R(0{,}0)$, $\gamma_x=-R^{(2,0)}(0{,}0)$, $\gamma_y=-R^{(0,2)}(0{,}0)$ $(R^{(2,0)}(0{,}0)=d^2R(x,y)/dx^2|_{(0.0)}=0$ and $R^{(0,2)}(0{,}0)=d^2R(x,y)/dy^2|_{(0.0)}=0)$. Assuming the response function is circular symmetrical, $\gamma_x=\gamma_y=\gamma$ [20].

Inserting the trial function into Eq. (2) and integrating Lagrange density over x, y, we obtain the average Lagrange

$$L=-2^{n_1+m_1}n_1!m_1!A_1^2\pi[(n_1+m_1+1)a_1^4\frac{dc_1}{dz}$$
$$+a_1^2\frac{d\theta_1}{dz}+\mu(n_1+m_1+1)(1+4c_1^2a_1^4)]$$
$$+\frac{1}{2}\rho\pi^2 2^{2(n_1+m_1)}(n_1!m_1!)^2A_1^4[R_0a_1^4-\gamma a_1^6(n_1+m_1+1)]$$
$$-2^{n_2+m_2}n_2!m_2!A_2^2\pi[(n_2+m_2+1)a_2^4\frac{dc_2}{dz}+a_2^2\frac{d\theta_2}{dz}$$
$$+\mu(n_2+m_2+1)(1+4c_2^2a_2^4)]$$
$$+\frac{1}{2}\rho\pi^2 2^{2(n_2+m_2)}(n_2!m_2!)^2A_2^4[R_0a_2^4-\gamma a_2^6(n_2+m_2+1)]$$
$$+\rho\pi^2 2^{n_1+m_1}n_1!m_1!2^{n_2+m_2}n_2!m_2!A_1^2A_2^2a_1^2a_2^2[R_0$$
$$-\frac{1}{2}\gamma(n_1+m_1+1)a_1^2-\frac{1}{2}\gamma(n_2+m_2+1)a_2^2] \quad (5)$$

The evolution equations for the parameters of the optical beams can be obtained based on the variational approach

$$A_j^2a_j^2=A_{j0}^2a_{j0}^2=\frac{P_{j0}}{2^{n_j+m_j}n_j!m_j!\pi} \quad (6a)$$

$$\frac{da_j}{dz}-4\mu c_ja_j=0 \quad (6b)$$

$$\frac{dc_j}{dz}=\frac{\mu}{a_j^4}-4c_j^2\mu-\frac{1}{2}\rho\gamma P_{j0}-\frac{1}{2}\rho\gamma P_{(3-j)0} \quad (6c)$$

$$\frac{d\theta_j}{dz}=-\frac{2\mu(n_j+m_j+1)}{a_j^2}+\rho R_0P_{j0}-\frac{1}{2}\rho\gamma P_{j0}a_j^2(n_j+m_j+1)$$
$$+\rho R_0P_{(3-j)0}-\frac{1}{2}\rho\gamma P_{(3-j)0}a_j^2(n_{3-j}+m_{3-j}+1) \quad (6d)$$

where P_{j0} $(j=1, 2)$ are the initial powers, a_{j0} and A_{j0} are the initial beam widths and amplitudes, respectively. The evolution equations of the beam widths can be obtained by combining Eqs. (6b) and (6c)

$$\frac{d^2a_j}{dz^2}=\frac{4\mu^2}{a_j^3}-2\mu\rho\gamma a_j(P_{j0}+P_{(3-j)0}) \quad (7)$$

where $P_0=P_{10}+P_{20}$ is the total incident power. So the evolution of such HG beam pairs depends only on the total initial power. The critical power of stationary soliton can be obtained by setting $d^2a_j/dz_j^2|_{z=0}=0$

$$P_{cj}=\frac{1}{k^2\gamma a_{j0}^4\eta} \quad (8)$$

It is obvious that when the total initial power is equal to the two critical powers, i.e., $P_0=P_{c1}=P_{c2}$, the two HG beams will both preserve their widths as they travel in the straight path along the z-axis. Namely the stable HG Vector soliton is formed. Furthermore, we can obtain $a_{10}=a_{20}$ by $P_{c1}=P_{c2}$. This is another necessary condition of forming a HG Vector soliton.

We normalize Eq. (7) by making

$$w_j=\frac{a_j}{a_{10}},\ Z=\frac{z}{ka_{10}^2} \quad (9)$$

where w_j is the normalized beam width, $Z=z/ka_{10}^2$ is the normalized Propagation Distance. Eq. (7) can be deduced to

$$\frac{d^2w_j}{dZ^2}=\frac{1}{w_j^3}-w_j\frac{P_0}{P_{cj}} \quad (10)$$

Assuming $w_j(0)=1$, $dw_j/dZ|_{Z=0}=0$, we can obtain the analytical solution by Eq. (10) and then compare the solution with numerical simulation.

3. Numerical results

For confirming the predictions of the variational approach, the split-step Fourier transform method will be employed to numerically simulate the propagation of HG beam pairs in strong nonlocal media. We use the approximate solution resulting from the variational approach as the initial condition and assume that the response function of the material is Gaussian-shaped, i.e., $R(x, y)=(1/\pi\sigma^2)\exp[-(x^2+y^2)/\sigma^2]$, where σ is the characteristic length of the material response function. Therefore $\alpha=\sigma/a_{10}$ represents the degree of nonlocal. For the strong nonlocal case, we have $\alpha\geq 10$. The normalized variables are given as: $Y=y/a_{10}$, $X=x/a_{10}$ are the normalized coordinates in the transverse direction, and $W_{j0}=a_{j0}/a_{10}$ $(j=1, 2)$ are the normalized initial beam widths.

3.1. The HG vector breather

The top and second row of Fig. 1 show that when the total initial power is smaller than the critical power, i.e., $P_0<P_c$, the two beams both expand initially which means that the diffraction effect initially overcomes the nonlinear effect. Therefore, the HG vector breather is formed as seen from the third row of Fig. 1. Likewise, the propagation of coupled HG beam pairs in the case of $P_0>P_c$ can be demonstrated numerically in Fig. 2, and the numerical results indicate that the two beams both contracted initially, which means that the nonlinear effect is stronger than diffraction effect.

The Fig. 3 displays the evolution lines of two beam widths. The comparison of analytical solution (solid lines) with numerical solution (dashed lines) shows that the analytical HG vector breathers are in good agreement with the numerical simulation in the case of strong nonlocality.