Optics Communications 322 (2014) 109–113

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Regions of spreading of partially coherent beams propagating through non-Kolmogorov turbulence

Xiaoling Ji [a,*], Zhengcai Pu [a], Guangming Ji [b], Xinhong Jia [a]

[a] Department of Physics, Sichuan Normal University, Chengdu 610066, China
[b] College of Information Management, Chengdu University of Technology, Chengdu 610059, China

ARTICLE INFO

Article history:
Received 7 November 2013
Received in revised form
2 January 2014
Accepted 8 February 2014
Available online 20 February 2014

Keywords:
Regions of spreading
Partially coherent beam
Non-Kolmogorov turbulence

ABSTRACT

In this paper, the regions of spreading of partially coherent beams propagating through non-Kolmogorov turbulence are examined by using the turbulence distances z_T and z_{TT}. The spreading is less affected by atmospheric turbulence (AT) when $z<z_T$, while the spreading is dominated by AT when $z>z_{TT}$. The dependence of z_T and z_{TT} on the turbulence parameters and the beam parameters is studied in detail. It is found that both z_T and z_{TT} reach their minimums when the fractal constant of the atmospheric power spectrum is equal to 3.11. However, the dependence of z_{TT} on turbulence parameters and beam parameters is much larger than that of z_T. There exists a minimum of z_T versus the waist width, but z_{TT} decreases monotonically as the waist width increases. On the other hand, the relation between the turbulence distance z_T and the Rayleigh range z_R is investigated. It is shown that $z_T=z_R$ may appear when a suitable waist width or a suitable coherent parameter is chosen. For the small free-space diffraction (FD) case $z_T<z_R$ may hold, but for the large FD case it may be $z_T>z_R$. Some physical explanations for the results obtained in this paper are also given.

1. Introduction

It has been experimentally shown that generally atmospheric turbulence might possess a structure different from the conventional Kolmogorov's one [1–3]. Such deviations are usually pertinent to higher atmospheric layers, being caused by gravity waves and jet-streams. Toselli et al. introduced a non-Kolmogorov power spectrum by using a generalized exponent and a generalized amplitude factor [4,5], and this non-Kolmogorov power spectrum reduces to the conventional Kolmogorov spectrum when the fractal constant of the atmospheric power spectrum is equal to 11/3. Baykal et al. studied the equivalence of structure constants in non-Kolmogorov and Kolmogorov spectra [6]. Recently, different propagation properties of different beams through non-Kolmogorov turbulence were studied [7–11]. Very recently, we investigated the influence of polychroism and decentration on spreading of laser beams propagating in non-Kolmogorov turbulence [12].

The turbulence induced spatial broadening of laser beams is a limiting factor in most applications. In recent years, much work has been carried out concerning the spreading of fully and partially coherent laser beams propagating through atmospheric turbulence. It has been shown that partially coherent beams are less sensitive to turbulence than fully coherent ones [13–15].

The region of spreading of partially coherent beams propagating through non-Kolmogorov turbulence is of considerable theoretical and practical interest. The Rayleigh range is used in the theory of lasers to characterize the distance over which a beam may be considered effectively non-spreading [16]. It has been shown that a partially coherent beam will have a Rayleigh range that is shorter than that of a fully coherent beam with the same intensity distribution in the beam waist [17]. In 2010, based on the effective radius of curvature, we extended the concept of the Rayleigh range of partially coherent beams from free space to turbulence [18]. The effective Rayleigh range of Gaussian array beams propagating through atmospheric turbulence was studied in Ref. [19]. In 2012, we examined two types of definitions for the Rayleigh range [20], where the two types of Rayleigh range were defined by the mean-squared beam width and the effective radius of curvature. On the other hand, in 2011, based on the relative beam propagation factor in turbulence, the range of turbulence-negligible propagation of laser beams was studied [21]. Very recently, under non-Kolmogorov turbulence, the range of turbulence-negligible propagation of laser beams was also examined [22].

In this paper, based on the mean-squared beam width, we apply the two parameters (i.e., turbulence distances z_T and z_{TT}) to indicate to what extent the beam spreading is less affected

* Corresponding author.
E-mail address: jiXL100@163.com (X. Ji).

http://dx.doi.org/10.1016/j.optcom.2014.02.032
0030-4018

by atmospheric turbulence (AT), and to what extent the beam spreading is dominated by AT. The dependence of the spreading regions on the turbulence parameters and the beam parameters is studied in detail. On the other hand, the relation between the turbulence distances z_T and the Rayleigh range z_R is investigated. Some physical explanations for the results obtained in this paper are also given.

2. Regions of spreading

2.1. Formulation

In this paper, the Gaussian Schell-model (GSM) beam is taken as a typical example of partially coherent beams. The cross-spectral density function of GSM beams at the source plane $z=0$ is expressed as

$$W^{(0)}(x_1', x_2', z=0) = \exp\left[-\frac{(x_1'^2 + x_2'^2)}{w_0^2}\right]\exp\left[-\frac{(x_1' - x_2')^2}{2\sigma_0^2}\right], \quad (1)$$

where σ_0 and w_0 are the spatial correlation length and waist width of GSM beams at the plane $z=0$, respectively.

Based on the extended Huygens–Fresnel principle, the average intensity of GSM beams propagating through non-Kolmogorov turbulence reads as [23]

$$\langle I(x,z)\rangle = \frac{k}{2\pi z}\iint dx_1'\, dx_2'\, W^{(0)}(x_1', x_2', z=0)\exp\left\{\frac{ik}{2z}[(x_1'2 - x_2'2) - 2(x_1' - x_2')x]\right\} \times\langle \exp[\psi^*(x, x_1', z) + \psi(x, x_2', z)]\rangle_m \quad (2)$$

where k is the wave number related to the wavelength λ by $k=2\pi/\lambda$, $\psi(x', x, z)$ represents the random part of the complex phase of a spherical wave due to the turbulence, $\langle\rangle_m$ denotes average over the ensemble of the turbulent medium, and [23]

$$\langle \exp[\psi^*(x, x_1', z) + \psi(x, x_2', z)]\rangle_m = \exp\left\{-4\pi^2 k^2 z\int_0^1\int_0^\infty \kappa\Phi_n(\kappa,\alpha)[1 - J_0(\kappa\xi|x_2' - x_1'|)d\kappa\, d\xi]\right\}, \quad (3)$$

where J_0 is the Bessel function of the first kind and order zero. $\Phi_n(\kappa,\alpha)$ is the power spectrum of fluctuations in the refractive index of the turbulent medium, κ is the magnitude of spatial wave number, and α is the fractal constant of the atmospheric turbulence power spectrum. In this paper, we assume that the turbulence is governed by non-Kolmogorov statistics, and that the power spectrum has the van Karman form, i.e. [4]

$$\Phi_n(\kappa,\alpha) = A(\alpha)\tilde{C}_n^2\frac{\exp[-(\kappa^2/\kappa_m^2)]}{(\kappa^2+\kappa_0^2)^{\alpha/2}}, \quad 0 \le \kappa < \infty, \quad 3 < \alpha < 4 \quad (4)$$

where $\tilde{C}_n^2$ is a generalized refractive-index structure parameter with units $m^{3-\alpha}$; $\kappa_0 = 2\pi/L_0$, L_0 is the turbulence outer scale; $\kappa_m = c(\alpha)/l_0$, l_0 is the turbulence inner scale; $A(\alpha) = [1/(4\pi^2)]\,\Gamma(\alpha-1)\cos(\alpha\pi/2)$, $c(\alpha) = [2\pi\Gamma(5/2-\alpha/2)A(\alpha)/3]^{1/(\alpha-5)}$, and Γ is the Gamma function. When $\alpha = 11/3$, we have $A(11/3) = 0.033$, and $\tilde{C}_n^2 = C_n^2$, and the spectrum reduces to the conventional Kolmogorov spectrum.

The mean-squared beam width is defined as

$$w^2(z) = \frac{4\int x^2\langle I(x,z)\rangle dx}{\int\langle I(x,z)\rangle dx} \quad (5)$$

On substituting from Eqs.(1)–(3) into Eq. (5), after very tedious integral calculations, we obtain the mean-squared beam width of GSM beams propagating in non-Kolmogorov turbulence, i.e.

$$w^2(z) = A + Bz^2 + Tz^3, \quad (6)$$

where $A = w_0^2$, $B = 4(1+1/\beta^2)/(k^2 w_0^2)$, $T = (8\pi^2/3)\int_0^\infty \kappa^3\Phi_n(\kappa,\alpha)d\kappa$, and $\beta = \sigma_0/w_0$ is called the coherent parameter.

The first two terms in Eq. (6) represent the effect of diffractive spreading of GSM beams in free space, while spreading of GSM beams due to non-Kolmogorov turbulence is represented by the third term Tz^3 in Eq. (6). The curve of T versus the fractal constant α of the atmospheric turbulence power spectrum is plotted in Fig. 1, where $L_0 = 10$ m, $l_0 = 10$ mm and $\tilde{C}_n^2 = 5\times10^{-14}\ m^{3-\alpha}$ are adopted. From Fig. 1 it can be seen that T reaches a maximum when$\alpha = 3.11$. It means that turbulence induces spatial beam broadening the most when$\alpha = 3.11$.

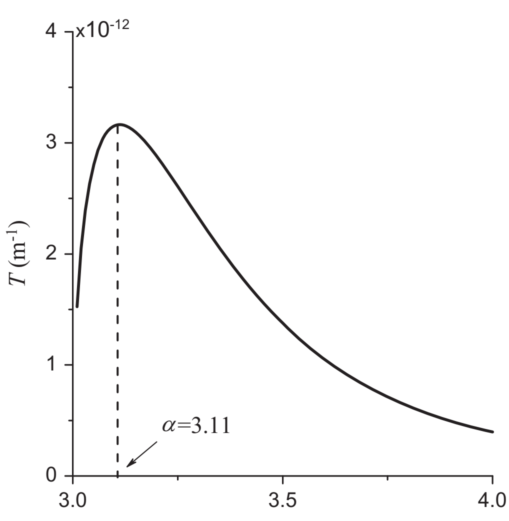

Fig. 1. Changes of T versus α.

2.1.1. Turbulence distances z_T and z_{TT}

The effects of the non-Kolmogorov turbulence on the spreading of GSM beams can be examined quantitatively by the turbulence distances z_T and z_{TT}. The turbulence distances z_T and z_{TT} are defined as

$$\frac{w^2(z_T) - w_{free}^2(z_T)}{w^2(z_T)} = \eta_1, \quad (7)$$

and

$$\frac{w^2(z_{TT}) - w_{free}^2(z_{TT})}{w^2(z_{TT})} = \eta_2, \quad (8)$$

where $w_{free}(z_T)$ and $w_{free}(z_{TT})$ are the mean-squared beam width in free space at $z=z_T$ and $z=z_{TT}$ planes, respectively, η_1 is a small positive constant (e.g., $0 < \eta_1 \le 10\%$), and η_2 is a large positive constant (e.g., $90\% \le \eta_2 < 100\%$). It is clear that the turbulence distance represents the distance at which the spreading due to the turbulence accounts for η (e.g., η_1 for z_T and η_2 for z_{TT}) of the cross-sectional area of GSM beams. The beam spreading is less affected by AT when $z < z_T$, while the beam spreading is dominated by AT when $z > z_{TT}$. It should be mentioned that the turbulence distance z_T for the $\eta_1 = 10\%$ case is defined in Ref. [13].

From Eqs. (6) and (7), we can obtain the cubic equation which determines z_T of GSM beams propagating through non-Kolmogorov turbulence, i.e.,

$$(1-\eta_1)Tz_T^3 - B\eta_1 z_T^2 - A\eta_1 = 0, \quad (9)$$

among the three solutions of this cubic equation, there is only one real solution, which represents the turbulence distance z_T, i.e.

$$z_T = \frac{G + \eta_1^2B^2/G + \eta_1 B}{3(1-\eta_1)T}, \quad (10)$$

where$G = \{27\eta_1(1-\eta_1)^2AT^2/2 + \eta_1^3B^3 + 3(1-\eta_1)T[81\eta_1^2(1-\eta_1)^2$