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Analytic solution to field distribution in one-dimensional inhomogeneous media

Tatjana Gric [a,b,c,*], Michael Cada [a]

[a] Dalhousie University, Halifax, Canada
[b] Semiconductor Physics Institute, Center for Physical Sciences and Technology, Vilnius, Lithuania
[c] Vilnius Gediminas Technical University, Vilnius, Lithuania

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ABSTRACT

A new general expression of a solution to Maxwell's equations derived recently has been applied to a 1-D inhomogeneous medium. It is shown that it solves any inhomogeneous refractive index profile. Its main advantage is that it does not require integration of either the differential wave equation or the refractive index profile, as it is the case with other methods. The solution is expressed in a closed form. The obtained numerical results compare favorably with both the slow-varying envelope method as well as with the exact numerical method.

1. Introduction

Photonic device design often involves light propagation studies in different structures with varying material properties, including inhomogeneous materials and optical waveguides. Traditionally, either approximate partially analytic methods or entirely numerical approaches have been used. Especially in device design a deep insight into light propagation properties is beneficial if an analytic method is available and can be applied. In practice however, one has to resort quite often to numerical approaches as analytical solutions are limited to just a few situations.

There is a number of interesting results and analytical approaches published that relate to the problem discussed here. The investigation of inhomogeneous waveguides is presented in [1–8]. A theoretical study of waves in a circular-cylindrical radially inhomogeneous guiding medium is proposed in [1]. A vector theory based upon Maxwell's equations is used to derive linear homogeneous fourth-order equations satisfied by the longitudinal electric and magnetic field components for a medium in which the permittivity decreases monotonically from the propagation axis.

Uniform waveguides filled with inhomogeneous dielectric whose permittivity varies along one dimension are studied in [2]. Emphasis is given to the modes of propagation and to the calculation of the propagation constants. A novel approach based on Asymptotic Iteration Method is presented in [3] to solve analytically the light propagation through a 1-D inhomogeneous slab waveguide. An approach based on Nikiforov–Uvarov method is presented in [4] to solve the light propagation through a 1-D inhomogeneous slab waveguide analytically.

A new set of exact solutions for the propagating modes in slab waveguides is obtained in [5]. Using the approach proposed, one can obtain a large class of inhomogeneous media with position-dependent permittivity and permeability with exact solutions for TM modes. The general solution is obtained in [6] of the 1-D wave equation of electrodynamics of inhomogeneous, anisotropic media in the form of a converging matrix series (an exponential integral). Maxwell's equations are solved for an inhomogeneous medium in [7], which has a coordinate-dependent dielectric function.

The Ermakov equation is derived from Maxwell's equations in [8] for inhomogeneous transparent media for 1-D cases. The plasma waveguides are investigated in [9,10]. The problem of strong magnetic field is solved in a 1-D non-uniform plane plasma waveguide in [10]. The propagation of an electrostatic wave in a non-uniform relativistic warm plasma waveguide under the effect of a high-frequency electric field is investigated in [10]. A new mathematical technique called "separation method" is applied to the two-fluid plasma model to separate the equations, which describe the system, into two parts, time and space. A wave approach is used in the analysis of wave motions in 1-D non-uniform waveguides in [11].

Maxwell's equations in their usual form provide a solution with a general time variation propagating in a loss-free medium. However, for a lossy medium, an attempt to obtain a general solution, which satisfies specified boundary and initial conditions,

* Corresponding author.
E-mail address: Tatjana.Gric@dal.ca (T. Gric).

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leads to divergences. It has been shown in [12] that modifying Maxwell's equations by adding an extra term, referred to as the “fictitious” magnetic charge density, allows one to obtain general solutions, with the boundary and initial conditions satisfied, in a lossy medium without encountering any divergencies. Maxwell's equations formulated for media with gradually changing conductivity are reduced to Volterra integral equations in [13].

Analytical and numerical solutions have been obtained in [14] for nonlinear waveguide structures with Kerr-like nonlinearity and linear homogeneous media. Three techniques are proposed for investigating the propagation characteristics of TE nonlinear surface waves propagating along a single interface of a nonlinear–inhomogeneous (single periodic layer) dielectric structure.

A new general formula that expresses the electric (or the magnetic) field vector that satisfies general vectorial time-dependent Maxwell's equations as a summation of time integrals and space derivatives has been derived in [15]. It is completely general and it is applicable to time-varying, inhomogeneous, nonlinear vectorial fields. It yields correct results for known analytic solutions and in some cases offers an attractive method of finding solutions without necessity of space integration. To our knowledge, this form has not so far been known or published elsewhere. In this paper we show its applicability and usefulness in expressing solutions to 1-D inhomogeneous cases analytically without necessity to integrate whatsoever. The 1-D problem is considered on purpose in order to demonstrate clearly that our new solution describes the situation correctly in a closed form and the results can be compared with those obtained by others (e.g. [16]) using different independent methods. However presented expression allows us to deal only with the 1-D problems. The investigation of 2-D optical waveguides [17] is considered as the future work.

The new, interesting and important results presented here is that our solution does not require cumbersome time-consuming integration and does not have to resort to performing numerical evaluations or employing analytical approximations, as is the case in many practical problems. The results are novel and original. The objective of this paper is to focus on the fundamental, mathematical aspects of the new solution presented and verified in [15] as it applies to practical cases of 1-D inhomogeneous media.

2. General solution for 1-D inhomogeneous media

Considering a 1-D problem, the dielectric constant of a medium varies only in the x-direction, which is basically a simple geometry of the problem studied. Maxwell's equations in the SI units for non-magnetic non-dispersive media are usually written as

$$\begin{aligned}
&\nabla \times \overline{E} = -\mu_0 \frac{\partial \overline{H}}{\partial t} \\
&\nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t} + \overline{J} \\
&\nabla \cdot \overline{H} = 0 \\
&\nabla \cdot \overline{D} = \rho,
\end{aligned} \tag{1}$$

with all the symbols known notoriously throughout the literature. The displacement vector is usually written out as $\overline{D} = \varepsilon_0 \overline{E} + \overline{P}$, with the polarization vector being expressed containing the linear medium (ε) as well as the possible nonlinear polarization ($\overline{P}_{NL}$) via material susceptibilities $\overline{\overline{\chi}}$'s as follows:

$$P = \varepsilon_0 \lfloor \overline{\overline{\chi}}^{(1)} \overline{E} + \overline{\overline{\chi}}^{(2)} \overline{E}\,\overline{E} + \overline{\overline{\chi}}^{(3)} \overline{E}\,\overline{E}\,\overline{E} + \ldots \rfloor = (\varepsilon_r \varepsilon_0 - \varepsilon_0)\overline{E} + \overline{P}_{NL} \tag{2}$$

It should be mentioned, that we are dealing with the linear medium. Thus $\overline{P}_{NL} = 0$. Moreover, we have to stress, that $\varepsilon_r = \varepsilon_r(x)$.

The basic derivation of our new solution is described in detail in [15]. Here we present only the final result for the electric field. For simplicity, we introduce the following symbols:

$$\int_t dt \cdots \;\; m-\text{times} = \int_t^m; \quad \text{and} \quad \nabla \times \cdots \;\; m-\text{times} = \nabla_m \times \cdots \tag{3}$$

With these notations, one writes the new solution as follows:

$$\begin{aligned}
\overline{E} = -\frac{1}{\varepsilon_0}\Big[\overline{S} - c^2 \int_t^2 \nabla_2 \times \overline{S} + c^4 \int_t^4 \nabla_4 \times \overline{S} - c^6 \int_t^6 \nabla_6 \\
\times \overline{S} + \cdots + c^m \int_t^m \nabla_m \times \overline{S} - \cdots\Big]
\end{aligned} \tag{4}$$

where m is even, and c is the speed of light in vacuum. S is a so-called source function that can be either independent of the solved electric field (left-hand side above) or it can contain the field itself, thus making the solution being implicit and recursive.

We now apply the solution in Eq. (4) to a 1-D inhomogeneous case thus obtaining

$$E = -\frac{1}{\varepsilon_0}\left[S - c^2 \int_t^2 \frac{\partial^2 S}{\partial x^2} + c^4 \int_t^4 \frac{\partial^4 S}{\partial x^4} - c^6 \int_t^6 \frac{\partial^6 S}{\partial x^6} + \cdots + c^m \int_t^m \frac{\partial^m S}{\partial x^m} - \cdots\right]. \tag{5}$$

Let us assume that the solution of Eq. (5) depends upon the time as (harmonic fields)

$$\overline{E} = [0, E_y, 0] \times e^{-j\omega t}. \tag{6}$$

Suppose that the source function S is expressed as follows [18]:

$$S = f(x, \beta) = 1 + a_1 x^2 + a_2 x^4 + a_3 x^6 + \cdots + a_n x^{2n} + a_{n+1} x^{2(n+1)} + a_{n+2} x^{2(n+2)} + \cdots, \tag{7}$$

where

$$\begin{cases}
a_1 = -A_0/2 \\
a_2 = A_0^2/24 \\
a_3 = -A_0^3/720 \\
\cdots \\
a_n = (-1)^n A_0^n/(2n!)
\end{cases}, \tag{8}$$

β is the propagation constant.

We have chosen the above-presented expression of the source function because we are considering the symmetric profiles that have been studied by others and thus can be mutually compared.

In order to find A_0 one needs to substitute Eq. (7) into Eq. (5). The derivatives are evaluated as

$$S'' = 2a_1 + 12a_2 x^2 + 30a_3 x^4 + 56a_4 x^6 + \cdots \tag{9a}$$

$$S^{IV} = 24a_2 + 360a_3 x^2 + 1680a_4 x^4 + \cdots \tag{9b}$$

$$\begin{aligned}
S^{2n} = (2n)!a_n + (2n+2) \times (2n+1) \times 2n \times \cdots \times 3x^2 a_{n+1} \\
+ (2n+4) \times (2n+3) \times (2n+2) \times (2n+1) \\
\times 2n \times \cdots \times 5x^4 a_{n+2} + \cdots
\end{aligned} \tag{9c}$$

$$\begin{aligned}
S^{2(n+1)} = (2(n+1))!a_{n+1} + (2n+4) \times (2n+3) \times (2n+2) \\
\times (2n+1) \times 2n \times \cdots \times 3x^2 a_{n+2} + \cdots
\end{aligned} \tag{9d}$$

After evaluating the time integration and space derivations in Eq. (5) a series is obtained:

$$\begin{aligned}
E = -\frac{1}{\varepsilon_0}\Big[S - \frac{1}{k_0^2} 2a_1 S + \frac{1}{k_0^4} 24a_2 S - \cdots - \frac{1}{k_0^{2n}} (2n)!a_n S \\
+ \frac{1}{k_0^{2(n+1)}} (2(n+1))!a_{n+1} S - \cdots\Big]
\end{aligned} \tag{10}$$

where $k_0 = \omega/c$. The substitution of the coefficients in Eq. (8) into expression (10) requires some rather cumbersome algebraic