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Discussion

# Generation and entanglement concentration for electron-spin entangled cluster states using charged quantum dots in optical microcavities

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ABSTRACT

We present schemes for deterministically generating multi-qubit electron-spin entangled cluster states by the giant circular birefringence, induced by the interface between the spin of a photon and the spin of an electron confined in a quantum dot embedded in a double-sided microcavity. Based on this interface, we construct the controlled phase flip (CPF) gate deterministically which is performed on electron-spin qubits and is the essential component of the cluster-state generation. As one of the universal gates, the CPF gate constructed can also be utilized in achieving scalable quantum computing. Besides, we propose the entanglement concentration protocol to reconstruct a partially entangled cluster state into a maximally entangled one, resorting to the projection measurement on an ancillary photon. By iterating the concentration scheme several times, the maximum success probability can be achieved. The fidelities and experimental feasibilities are analyzed with respect to currently available techniques, indicating that our schemes can work well in both the strong and weak (Purcell) coupling regimes.

## 1. Introduction

Under the motivation to develop large-scale quantum algorithms, a revolutionary computing paradigm, i.e. measurement-based quantum computation (MBQC), was introduced by Raussendorf and Briegel [1], where quantum computation can be performed by implementing local measurements on highly entangled quantum states. Apart from the alternative Affleck–Kennedy–Lieb–Tasaki (AKLT) states [2], cluster states have been considered as the most promising candidate to be the universal resource due to its high connectedness and large persistency of entanglement. Additionally, they can be utilized as a resource for generating other multi-qubit entangled states. Thus, a wide range of quantum information processing (QIP) proposals using cluster states have been proposed [3]. Meanwhile, of particular interest is the generation of cluster states in a variety of systems, including linear optics [4], where the scalability remains a stumbling block due to the need to generate the initial multi-qubit photonic entanglement through, for example, concatenating parametric down conversion process, and cavity QED [5], which often requires the system to operate in the strong-coupling regime, which means the vacuum Rabi frequency of the dipole exceeds both the cavity and the dipole decay rates. The scheme that generates cluster states using neutral atoms in optical lattices [6] has also been proposed, which remains to be difficult concerning the lack of individual addressing. Recently, Hu et al. also mulled over the possibility to perform QIP based on endohedral fullerene systems, proposing the direct and indirect methods, respectively, to generate cluster states with arrays of endohedral fullerenes residing in single-walled carbon nanotubes [7]. On the other hand, there have recently been theoretical and experimental breakthroughs into the hybrid quantum system, consisting of a singly charged electron confined in a quantum dot (QD) inside an optical resonant microcavity and the ancillary photons to be the flying qubits. This system has attracted extensively exploration and is considered as a viable candidate for QIP tasks. Within this system, incorporating QDs into solid-state cavities is comparatively easy, which largely enhances its scalability and stability. Besides, the electron-spin coherence time is rather long that has been prolonged to μs range using spin-echo techniques [8–11], which greatly promotes the feasibility of realizing large numbers of unitary operations in QIP. Additionally, the initialization of electron-spin states can be realized by means of optical pumping or optical cooling [12], where, in the case of superposition states, ultrafast optical rotation of the single spin can be employed [13]. The spin-state preparation with a fidelity

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exceeding 99.8% was reported by Press et al. [14]. Ultrafast coherent control over the electron-spin states using picosecond optical pulses were reported, so that a large number of spin rotations could be carried out while the coherence is maintained [13,15–18]. Efficient optical methods for readout of spins in QDs have been developed in recent years [19–21]. In 2008 and 2009, Hu et al. proposed three schemes to entangle remote spins [22], photons [23] and to realize entanglement beam splitter [24], respectively, using the coupled system of QD spins and optical resonant cavities. Subsequently, a variety of technology have been employed resorting to this spin-QD-cavity system, including the constructions of universal hybrid quantum gates, namely CNOT gate, Fredkin and Toffoli gates [25,26]. In 2013, Deng et al. proposed the deterministic two-qubit and three-qubit universal quantum gates on photonic circular-polarization qubits [27]. Schemes for efficient entanglement teleportation, swapping [28], concentration and purification [29,30] of electron-spin entangled states and quantum repeaters [31] were presented in 2011 and 2012, respectively. Recently, two proposals concerning the generation and complete analysis of hyper-entangled Bell states for photons were addressed assisted by the electron-spin interface in this system [32].

In addition, for utilizing entanglement in real QIP, it is mandatory to take into account the decoherence resulting from the coupling of quantum systems to the environment, which destroys the fragile quantum information rapidly. Two comprehensive methods that depress the effect of noise are quantum error-correction code (QECC) [33] and quantum fault tolerant operations [34–36]. However, the entire framework of quantum fault tolerance and the encoding, decoding processes of QECC promise to be comparatively difficult and expensive in practice in terms of the number of ancillary qubits required and the number of physical gates implemented, especially the controlled gates. There exist yet two other more appealing methods when we are concerned with recovering a subset of shared entanglement from a larger number of shared but less entangled states between distant parties. Here, when the initial shared but less entangled states are mixed, this recovery of entanglement is termed as entanglement purification or distillation [37]. Alternatively, entanglement concentration copes with the cases that distilling maximally entangled states from less entangled pure states. The importance of such a scheme is obvious as maximally entangled states are essential for various QIP with perfect fidelity. In 1996, C. H. Bennett proposed the original entanglement concentration protocol (ECP) [38] (called Schmidt projection method). The achievement of this proposal requires the collective measurement for the joint state of $n$ pairs of particles, which is difficult to perform in practice, especially for long-distance quantum communication using photons. Subsequently, similar schemes, namely ECPs based on quantum swapping [39] and merely linear optical elements [40], were investigated independently where local measurements on two photons associated with classical communication realize the required projection. The corresponding experiments were reported later in 2008 [41]. To date, assorted breakthroughs have been made tackling entanglement concentration for Bell-class states, GHZ-class states as well as W states, such as the ones based on Nielsen's theorem [42] which realize concentration in a deterministic manner by performing positive-operator-valued measurement locally combined with classical communication, and the ones resorting to cavity QED [43], among which an ECP using photonic Faraday rotation that requires low-Q cavity was presented recently [44]. Various ECPs have been proposed with the help of cross-Kerr nonlinearity, where the nonlinear effect can be used to realize parity-check quantum nondemolition detection (QND) [45]. Moreover, three recent studies showed that ECP(EPP) can be performed more efficiently with a higher success probability(deterministically) when hyperentanglement is available [46–48]. For concentration, Ren et al. [48] proposed the parameter-splitting method to extract the maximally entangled photons in both the polarization and spatial degrees of freedom when the coefficients of the initial partially hyper-entangled state are known. This fascinating method is demonstrated to be very efficient and simple in terms of concentrating partially entangled state in one degree of freedom, especially a less-entangled polarization state. It can be achieved with the maximum success probability by performing the protocol only once, resorting to linear optical elements only. ECPs for electron-spin entangled Bell states [29,49] and W states [50] were presented, respectively, using quantum-dot spins inside optical microcavities. Currently, most of the protocols aim at concentration for entangled Bell-class, GHZ-class states or W states on photonic systems. However, concentration for cluster states on solid-state systems deserves to be developed in the regime of measurement-based quantum computation. Arguably, Choudhury et al. [51] reported the ECP for cluster states based mainly on the property of polarization beam splitter (PBS), where the success probability is relatively low and its postselection nature leads to the fact that once the required state is chosen, the composite state is detected and then destroyed. The ECP for cluster states with the help of cross-Kerr nonlinearity was also proposed [52]. However, it remains a challenge to initially achieve a high order of phase shift at the single photon level, even with electromagnetically induced transparency [53]. Besides, it is currently controversial whether these nonlinearities are sufficient for single-photon quantum applications [54]. The difficulties associated with these schemes prevent the realization and utilization of ECP for cluster states in large-scale quantum computation.

In this paper, we explore firstly the deterministic generation of electron-spin entangled cluster states using the hybrid system (spin-QD-cavity system). The double-sided cavity is preferred rather than the single-sided one due to its improved robust and flexible feature, i.e. the reflectance for the uncoupled and coupled cavity is not strictly required to be balanceable to obtain high fidelity [28]. The core of the scheme, namely the CPF gate, is constructed deterministically that is the essential building block in a host of QIP. Then, the concentration scheme for four-qubit cluster states is introduced, which is then be generalized to the multi-qubit cases. Within the protocol, merely one copy of the initial partially entangled state and one ancillary photon are required. The generation and concentration schemes are proved to work well in both the strong and weak(Purcell) coupling regimes.

The paper is organized as follows. In Section 2, we propose a scheme for implementing CPF gate deterministically on electron-spin qubits. Then, the generation for cluster states is discussed. In Section 3, we explain our concentration scheme in an ideal situation. The fidelities and experimental feasibilities from a practical point of view are discussed in Section 4, while some discussions and conclusions are shown in Section 5.

## 2. Schemes for generating cluster states in QD-cavity system

Consider a singly charged QD, e.g. a self-assembled GaAs/InAs or GaAs interface QD, placed in a double-sided optical resonant microcavity. As shown in Fig. 1, there are four relevant electronic levels and two optically allowed transitions of the trion $X^-$ (also called a negatively charged exciton), which are spin dependent due to the Pauli's exclusion principle. This optical property introduces large differences in the phase or the amplitude of the reflection and transmission coefficients between the two circularly polarized photons, one involving a $s_z = +1$ photon and the other involving a $s_z = -1$ photon. This giant circular birefringence works in both the strong and weak coupling regimes [25,28]. In detail, as Fig. 1 illustrates, the quantization axis for angular momentum is set along the normal direction of the cavity axis, that is, the $z$-axis.