Optics Communications 319 (2014) 159–169

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Doppler-spectrally encoded imaging of translational objects

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ARTICLE INFO

Article history:
Received 30 August 2013
Received in revised form
20 December 2013
Accepted 5 January 2014
Available online 22 January 2014

Keywords:
Doppler effect
Spectral encoding
Translational object
Phase imaging
Synthetic aperture tomography
Imaging flow cytometry

ABSTRACT

The image of a translational target is moved by the Doppler-shifted phase of the diffraction field of the light incident on the target. However, no one has yet utilized the physical relationship between the Doppler effect and the diffraction field in microscopic imaging. Here, we demonstrate Doppler-spectral encoding of the diffraction field of a translational target. We found that the angular spectrum of the translational object was encoded by the Doppler spectrum, and the interferometric recombination of the Doppler spectrum yielded a 2-dimensional complex image. We further discovered that two Doppler effects, which are evoked by the movement of the target against a stationary source and detector, can be exploited simultaneously in synthetic aperture tomography. Doppler-spectrally encoded imaging may lead not only to label-free imaging flow cytometry of living cells but also to non-destructive imaging of products during inspection on a conveyer belt in either the sound or electromagnetic regimes.

1. Introduction

If light interacts with a moving target, it can acquire a Doppler-shifted phase whose continuous change yields a Doppler-shifted frequency. This physical process holds true, for example, when perceiving one's own hand movement although the speed of that motion is in fact very slow compared with that of light. However, no one perceives the hand movement itself as resulting from the Doppler effect. Similarly, no one has exploited the Doppler effect to perform microscopic coherent imaging of translational objects. Hufnagel [1] first proposed in 1966 that rotating objects such as astronomical satellites can be imaged by exploiting the Doppler effect. Goodman [2] reviewed the topic, and the idea was later extended to the acoustic and microwave regimes [3]. Hufnagel referred to this method as “Doppler-spread imaging” and assumed that the light incident on the satellite returns back to the detectors specularly because the satellite is very far away. As a result, his technique detects the specular reflection field rather than the total diffraction field. In the case of objects that are close to the observer, a slit has been used to super-resolve the image by exploiting the Doppler-shifted light only from the specular reflections [4,5]. These techniques must utilize specific incident and scattering vectors because they require that the projection distance to the rotational axis is linearly related to the Doppler-shifted frequency. Such imaging techniques are useful in cases where the objects are far away or a low numerical aperture objective lens is facing the objects. A separate non-microscopic imaging technique utilizing the Doppler effect is known as synthetic aperture radar (SAR) (initially called “Doppler Beam Sharpening” (DBS) by C. A. Wiley in the 1950s). This technique also exploits the phase shift caused by the Doppler effect. In SAR, the experimenter records the amplitude and instantaneous phase shift $2kR$ (of the complex field), which signify the zone plate for a distance R between the scatterer and the airborne antenna as an aircraft approaches, passes, and recedes from the scatterer. The recorded complex field is the Fourier transform of an image of the scatterer. In the field of biological applications, spectral encoding techniques [6–8] artificially encode positions on the sample with wavelengths of light. Such techniques are used to perform reflectance imaging of translational objects such as cells flowing in micro-fluid devices or blood vessels by scanning the samples with a transverse spectrally encoded line beam. Yelin [9] measured 2-dimensional flow velocities across a tube by using the Doppler effect of *a priori* spectrally encoded light.

In contrast to *a priori* spectrally encoded imaging [6–9], we impinge a monochromatic light beam (not a line beam) that has a diameter that covers the translational object and detects the entire diffraction field, which consists of frequencies that are *a posteriori* Doppler-spectrally encoded, with a 2-dimensional camera. Here, we first introduce a Doppler spectrum representation of the diffracted field for the translational objects according to the Born approximation. Then, we perform temporal 2-dimensional coherent imaging of the translational objects using the Doppler spectrum representation. Finally, we perform synthetic-aperture tomography based on the Doppler spectrum representation for 3-dimensional, label-free imaging flow cytometry applications.

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http://dx.doi.org/10.1016/j.optcom.2014.01.013

2. Methods

2.1. Principle

The Doppler effect is evoked in two ways: once when the incident laser light of the stationary transmitter system impinges on the moving target and once when the light is scattered from the moving target and received by a stationary detector [10]. The Doppler-shifted frequency $\Delta\omega$ of the diffracted light from the translational object is therefore the inner product of the velocity $\boldsymbol{V}$ of the translational object and the difference vector of the incident light wave vector $\boldsymbol{k}_0$ and the scattering wave vector $\boldsymbol{k}$:

$$\Delta\omega = (\boldsymbol{k} - \boldsymbol{k}_0) \times \boldsymbol{V}. \tag{1}$$

For simplicity, we assume that the incident light $\boldsymbol{k}_0$ is normal to the translational direction of the object such that the right side of Eq. (1) represents the inner product of $\boldsymbol{k}$ and $\boldsymbol{V}$.

In the late 19th century, Ernst Abbe described an object as a fine grating in his theory of image formation in the microscopy [14], and all objects are composed of gratings having various spatial frequencies. Because an imaging system is a linear system, we can consider a grating with a single spatial frequency as an example. In Fig. 1, a sinusoidal grating whose spatial frequency is K_g along the x-axis is moving linearly along the x-axis at a constant velocity V. The collimated light depicted in green in Fig. 1 is normally incident on the translational grating. Because the first-order (± 1) diffracted light beams are frequency-shifted due to Doppler effect, these beams are depicted as blue and red while the zero-order light is the same color as the incident beam as it does not have any frequency shift. If k is the scalar of the wave vector $\boldsymbol{k}$, the scattering angles $\pm\theta$ of the first-order diffracted light beams are expressed as $\pm\sin^{-1}(K_g/k)$ [15]. The frequency shift $\Delta\omega$ of the first-order diffracted light is calculated as $\Delta\omega = \pm K_g V$ by substituting the scattering angle θ into Eq. (1). Here, we find that the spatial frequency K_g is linearly related to the Doppler-shifted frequency $\Delta\omega$ by the factor V. Fig. 1 also depicts that these frequency-shifted diffracted beams are focused on the Fourier plane in spatially separated locations by lens 1 and that the temporally different frequency-shifted beams are spatially recombined on the back focal plane (image plane) of lens 2 such that the focused image will move. Although temporal focusing [11–13] does not utilize the Doppler effect, the recombination of spatially separated frequencies of broadband light with a stationary grating is analogous to the recombination of spatially separated Doppler-shifted frequencies of monochromatic light with a translational grating.

So far, we have described an intuitive understanding of Doppler spectral encoding and decoding of an image of a translational object. Next, we introduce an integral equation for the scattering potential $F(\boldsymbol{r})$ in Eq. (2) to generate a rigorous relationship between the Doppler-shifted frequency and the diffracted field. We will base our analysis on the first-order Born approximation for the diffracted field $U_1^{(S)}$ of a stationary object at infinity.

$$U_1^{(S)}(\boldsymbol{r}) = \int_V U(\boldsymbol{r}')F(\boldsymbol{r}')G(\boldsymbol{r}-\boldsymbol{r}')d^3\boldsymbol{r}' = \frac{\exp(ikr)}{r}\int_V F(\boldsymbol{r}')\exp(-i\boldsymbol{K}\times\boldsymbol{r}')d^3\boldsymbol{r}', \quad \text{as} \quad kr\to\infty \tag{2}$$

where $G(\boldsymbol{r})$ is the Green's function $\exp(ik|\boldsymbol{r}-\boldsymbol{r}'|)/|\boldsymbol{r}-\boldsymbol{r}'|$, $\boldsymbol{K}$ is $k(\boldsymbol{s}-\boldsymbol{s}_0)$, and $F(\boldsymbol{r})$ is $k^2[n^2(\boldsymbol{r})-n_0^2(\boldsymbol{r})]/4\pi$ (n and n_0 are the refractive indices of the sample and the medium, respectively). $\boldsymbol{s}_0$ and $\boldsymbol{s}$ are the unit vectors of the incident and scattered light. This equation is well known, yet no one to our knowledge has utilized it to take the Doppler effect of a translational object into account [1–5]. We derived the diffracted field of a translational object by applying Galilean transformation [16] to that of a stationary object represented in Eq. (2). The first inertial frame $\boldsymbol{r}_1$, which includes the transmitter and receiver, now stands still while the second inertial frame $\boldsymbol{r}_2$, which includes the object, moves at a constant velocity $\boldsymbol{V}$. According to Galilean relativity, the space and time coordinates in the two frames are related by

$$\boldsymbol{r}_2 = \boldsymbol{r}_1 - \boldsymbol{V}t_1, \quad t_2 = t_1. \tag{3}$$

We can apply this transformation to Eq. (2) because the velocity of the object is small compared to the speed of light. Imagine that an incident field $U(\boldsymbol{r})$ is transmitted and a scattered field (field propagator) $G(\boldsymbol{r})$ is received in the first frame, and the scattering potential $F(\boldsymbol{r})$ is in the second frame. Substituting with Eq. (3) into the term $F(\boldsymbol{r})$ and changing variables yields the scattered field of a translational object:

$$U_{1\text{Doppler}}^{(s)}(\boldsymbol{r}) = \frac{\exp(ikr)}{r}\exp[-i(\omega_0 + K_xV_x + K_yV_y + K_zV_z)t]\tilde{F}(K_x, K_y, K_z) \tag{4}$$

where we added an original light frequency term $\exp(-i\omega_0 t)$. $\tilde{F}(\boldsymbol{K})$ is the Fourier transform of the scattering potential $F(\boldsymbol{K})$ and is known as the scattering amplitude in the far-field zone of the scatterer. It is very simply related to the angular spectrum representation [17]. In the case of a relatively thin object, the representation is the angular spectrum itself. In this sense, the diffracted field of the translational object at infinity consists of an angular spectrum whose phase term is temporally modulated by the Doppler-shifted frequency $\Delta\omega$ according to Eq. (5). In Fourier optics, it is well-known that the Fourier image of a translational object changes in the phase but not the amplitude. This fact is also mathematically known as a "phase (or time) shift" and is the one of the properties of the Fourier transform. The continuous change of the phase in the angular spectrum yields a change of the temporal frequency, which is called the Doppler-shifted frequency. In other words, the Doppler effect causes a conversion of spatial frequencies (angular spectrum) into temporal frequencies. Later, we will show that the angular spectrum of a translational object is frequency-shifted (colored) along the translational direction.

Eq. (4) implies that the angular spectrum of the scattered field $\tilde{F}(\boldsymbol{K})$ is encoded by a Doppler-shifted frequency $\boldsymbol{kV}$ (in simple nomenclature, the Doppler spectral encoding) if the incident light is normal to the translational direction of the object (i.e., $\boldsymbol{k}_0\boldsymbol{V}=0$). Furthermore, if the translation axis ($\boldsymbol{V}$) of the object is parallel to K_x (i.e., $V_y=V_z=0$), one can at once retrieve the k_x-axis component of the scattering potential $F(V_xt)$ in time from an integration of the scattered field with respect to k_x. This integration explains how the image of a translational object is focused. Considering that the act of focusing an image of a stationary object can be defined as the recombination of the angular spectrum of the object and that the angular spectrum $\tilde{F}(\boldsymbol{kV})$ is related to the temporal spectrum *exp* ($i\boldsymbol{kV}$) by the Doppler effect (Eq. (1)), the recombination of the

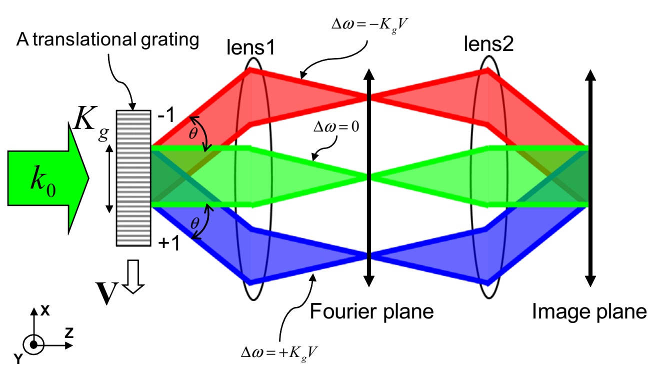

Fig. 1. *Principle for converting the spatial frequency of a translational grating to a temporal frequency by the Doppler effect.* The scattering angle θ is defined by $\sin^{-1}(K_g/k)$. k is the wavenumber, and K_g is the spatial frequency of the grating.