Optics Communications 319 (2014) 17–24

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Temporal modulation instability, transition to chaos in non-feedback biased photorefractive media

Morteza A. Sharif [a,*], Mehdi Borjkhani [b], Bijan Ghafary [a]

[a] Department of Physics, Iran University of Science & Technology, Narmak, Tehran, Iran
[b] Electrical Engineering Group, Urmia University of Technology, Band Road, Urmia, Iran

A R T I C L E I N F O

Article history:
Received 12 October 2013
Received in revised form
4 December 2013
Accepted 23 December 2013
Available online 7 January 2014

Keywords:
Modulation instability
Chaos
Lithium Niobate
Delayed feedback system
Nonlinear photorefractive medium′
Non-feedback chaos control

A B S T R A C T

This paper surveys the theoretical dynamic model of chaotic regime in optical delayed feedback system; chaotic control parameters of optical input intensity and externally applied bias electric field are investigated. It is also shown that quasi-periodic state identified as temporal modulation instability can be deeply considered as a route to chaos through the evolution equation. Numerical solution of nonlinear Schrödinger equation as the universal model of modulation instability approves such claim. Pre-experiment based on optical delayed feedback system confirms theoretical model results and clarifies the crucial role of critical frequency as the competition point between optical bistability and the chaotic regime. Then, the simple experiment of non-feedback chaos control in Lithium Niobate photorefractive medium without delay indicates that quasi-periodic state -implies on temporal modulation instability- is also attainable and thus chaotic control can be achieved. The causal explanation of such behavior in slow response time Lithium Niobate photorefractive medium is analytically discussed as the generation of the internal feedback inside the medium.

1. Introduction

Low threshold power and the possibility of controlling light with light are of the attraction features of photorefractive (PR) materials. Chaotic optics communication is indebted to convenient chaos control utilizing nonlinear PR media due to the intrinsic chaotic feature of charge carrier generation in accordance with radiant intensity of light. While a PR material is exposed to the optical beam, free charge carriers are generated by excitation from impurity energy levels to an energy band at a rate proportional to the optical power; forthcoming variation of these charge carriers density is depended on recombination rate, trap density and therefore, the time series of this variation reveals a stochastic and intermittent chaotic behavior. Overall, photo-induced charges create a space-charge distribution that produces an internal field (space-charge field), which in turn alters the refractive index by means of the electro-optic effect. Produced sub-lattice of nano/microstructure in illuminated region of photorefractive crystal can also change dielectric constant, conductivity and other parameters. Compared to the other nonlinear media (like Kerr medium), PR materials are known as nonlinear optical materials in which the nonlinear response is variable depended on nonlinear effects; nonlinearity is simply depended to the space-charge field generated by the optical beam; hence, the nonlinear behavior can be controlled by externally applied electric field [1–12].

On the other hand, modulation instability (MI) in a nonlinear medium is known as an optical nonlinear process in which small perturbations upon a uniform intensity beam grow exponentially due to the interplay between nonlinearity and dispersion. MI has been experimentally observed as the various dialectic forms of spatial MI, temporal MI, multiple pattern instability, transverse MI of soliton stripes, etc. Observations on both spatial and temporal MI include generated spectral sidebands due to the nonlinearity. The growth rate of these sidebands perturbations is proportional to photo-induced space-charge field. Whenever, the applied bias electric field on a nonlinear PR medium is increased, the generated sidebands number and amplitude will be also increased; the reason is known as the rising of MI gain [1,6,7,9,10,12–17]. On this base, the effect MI can be attributed to transition from stationary regime to chaotic regime. Perturbations and fluctuations can be stochastic or chaotic above a certain threshold of control parameter. Meanwhile, the character of MI can be converted from its convective to absolute form according to Ruelle–Takens scenario. Much below the *critical frequency*, the oscillations will be no more predictable and thus, the absolute MI can be ascribed as a route to chaos [14–17,25–28].

In spite of the multiple studies on theoretical description of MI and the manner MI gain rises- and accordingly the way the system transits to chaotic state, fewer attempts have been done to investigate the procedure from the view point of evolution equation. On the other

* Corresponding author. Tel.: +98 2177240477, fax: +984413554184.
E-mail addresses: m_a_sharif@iust.ac.ir,
m.abdolahisharif@ee.uut.ac.ir (M.A. Sharif),
m.borjkhani@ee.uut.ac.ir.(M. Borjkhani)ghafary@iust.ac.ir (B. Ghafary).

0030-4018/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.optcom.2013.12.064

side, many recent studies have investigated the control parameters interfered in spatiotemporal transition to chaos inside the nonlinear PR media; nevertheless, the question how the nonlinear dynamics evolves in externally biased PR medium is still remained. Moreover, the almost whole experimental observations use the conventional optical delayed feedback system (DFS) for a PR medium to provide an experimental setup of spatiotemporal transition to chaos—which needs a delayed feedback circuit to set the *critical frequency* whenever the delay time is coincident with PR slow response time. In order to get rid of delayed feedback circuit—which imposes own problems, this paper deals with a simple non-feedback optical setup without delay to achieve the same transition. Therefore, the question how a PR medium with such a slow response time and without any delayed feedback can undergo quasi-periodic oscillations identified as MI will be open [1,7,9,14,16–17,28]. Would the rare present experimental observations answer this question?

This paper is organized in the following sections: Section 2 surveys the theoretical dynamic model of chaotic regime in optical DFS; chaotic control parameters of optical input intensity and externally applied bias electric field and the correlation between them are investigated. It is declared that temporal MI can be construed as a prior to chaotic state by the means of evolution equation. Numerical solution of nonlinear Schrödinger (NLS) equation is also reconstructed to model MI. Afterward, pre-experiment based on optical DFS in Section 3, truly validates the theoretical model results elucidating the crucial role of *critical frequency* as the competition point between the optical bistability and the chaotic instability. The main Section 4 is dedicated to the experiment of non-feedback chaos control in Lithium Niobate PR medium without delay in which the observations show that MI can be achieved. In Section 5, the causal explanation of quasi-periodic oscillations intimated with MI in Lithium Niobate PR medium with slow response time is analytically discussed. Finally, Section 6 is devoted to the conclusion of the paper. Note that throughout the paper, the nonlinear medium is considered as a typical PR medium.

2. Theoretical description of nonlinear dynamics in DFS

2.1. Chaotic control parameter of optical input intensity

The nonlinear dynamics of an optical DFS can be theoretically described by evolution Eq. (1) [29–35]:

$$V(t)+\tau\frac{dV(t)}{dt}=\frac{1}{2}I_1\{1-\delta\cos[V(t-T)+\Delta\varphi]\}+X(t-T) \quad (1)$$

where I_1 and $V(t)$ represent the input and output intensity level to/from the optical nonlinear PR medium and $V(t)$ is feedback voltage; T is considered as the delay time in feedback loop, τ is the nonlinear PR medium response time, X is control weight scaled with feedback voltage, added when delayed feedback control of delay time T is assumed in experimental setup; $\Delta\varphi$ and δ are the phase shift and the modulation depth—which will be fully declared about in the following sections. Whenever the optical input intensity – which enters in optical PR medium – exceeds a certain threshold value, the stationary regime becomes unstable and self-modulation arises; with further increase in the intensity, a transition to chaos occurs in a complicated way and a sequence of period-doubling bifurcation which is a universal feature of nonlinear dynamics for the system transition from steady state to chaotic state, is then inevitable [18–35]. Eq. (1) can be numerically solved as given in Eq. (2) with assumption $0<t'\le T$, $t=(N-1)T+t'$, and $V_N(\tau)=V(t)$, and also $\tau dV(t)/dt\approx 0$, if $T\gg\tau$—i.e. the condition of much below the *critical frequency* [25,29–35]:

$$V_{N+1}(t')=\frac{XV_N(t')+\frac{1}{2}I_1\{1-\delta\cos[V_N(t')+\Delta\varphi]\}}{1+X} \quad (2)$$

The first Lyapunov exponent is recognized as:

$$\lambda_1=\max_{\{0<t'\le T\}}\left\{\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\ln\left|\frac{X+(1/2)I_1\delta\sin[V_N(t')+\Delta\varphi]}{1+X}\right|\right\} \quad (3)$$

Fig. 1 shows the output profile of optical DFS according to Eq. (2); the steady state periodic output profile (Fig. 1a) is evolved to the period-doubling output profile (Fig. 1b) and then to the complex chaotic profile (Fig. 1d) while the optical input intensity as a control parameter is increasing. A fully chaotic regime can thus occur if the system is modulationally unstable. Fast Fourier transform (FFT) of intensity-time diagram of output profile of Fig. 1b (Fig. 2a) reveals the first spectral sideband perturbation of temporal MI. Accordingly, higher order MI performs sidebands number and perturbations which increase and grow exponentially – while the central frequency remains intact except for its amplified amplitude – over frequency domain. All quantities of Figs. 1 and 2 are in arbitrary unit.

A fully detailed theoretical description of temporal MI will be presented in the following sections. Describe enough the conceivable consequence of such sidebands perturbations is the complex chaotic behavior—which is subsequently indicated in Fig. 2b and c as the first Lyapunov exponent and bifurcation diagram, both vs. the optical input intensity.

On the side, If $T\ll\tau$ is assumed in Eq. (1)—and thus, the condition of much below the *critical frequency* is not fulfilled, the numerical solution will not appear as Eq. (2). Instead, for extremely short delay time, the output profile will exhibit a sudden jump from one lower stationary state to the upper stable state – known as optical bistability – if the optical input intensity exceeds over the threshold value. Through the procedure, strong carrier wave grows and finally switches to the upper stable state while the sidebands gradually disappear. This is annotated as the competition between MI and the optical bistability [21,26–35].

2.2. Chaotic control parameter of external bias electric field

Chaos communication conventionally utilizes an electro-optical modulation (EOM) DFS [36–39]. The optical input beam passing through the nonlinear PR medium is exposed to an external bias voltage (and consequently an external bias electric field) which is fixed at optimum point of output characteristic curve (OCC) of EOM- and some alternative voltage (modulation voltage) fluctuates around this point [40]. Therefore, the output profile of EOM is depended on the external bias electric field. Considering the transverse modulation, the parameters $\Delta\varphi$ and δ of Eq. (1) can be written as Eq. (4):

$$\begin{cases}\Delta\varphi=\pi\frac{V_b}{V_\pi}\frac{L}{d}\\ \delta=\pi\frac{V_b}{V_\pi}\end{cases} \quad (4)$$

whereas L represents the length of the crystal, d, the width of it (i.e. the lateral separation of applying bias voltage electrodes), V_π is the half wave voltage and $E=V_b/d$ is the external bias electric field [40,41].

Intensity of light beam can be modulated and the transmittance of EOM without DFS as the ratio of output intensity to the input intensity $T=(V/I_1)$ can be rewritten as equation $T=\frac{1}{2}[1-\delta\cos\Gamma]$ $=(1/2)[1-\delta\cos(\Gamma_0+(\pi V_b/V_\pi))]$ where Γ_0 is a fixed value pertaining to phase retarder ($\Gamma_0=\pi/2$ for quarter-wave plate). If the bias voltage is very large, the output wave will be distorted and higher order odd harmonics will be generated as the sidebands in