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Negative phase velocity and negative refraction in orthorhombic dielectric–magnetic lossy media with polarization along one of principal axes

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ABSTRACT

The characteristics of the TE plane wave propagations in orthorhombic dielectric–magnetic lossy media are investigated. The special case for the electric field being parallel to one principal axis and the wave vector lying wholly in the plane formed by the other two principal axes is considered. The dissipation is taken into account by letting the elements of the permittivity and the permeability tensors to be complex. The general condition on the negative phase velocity for uniform TE plane waves is provided in terms of material parameters and the propagation angle. The numerical calculations for the angles between the average Poynting vector and the phase velocity are performed in order to justify the theoretical analysis. In addition, the TE refraction between the free space and the orthorhombic dielectric–magnetic lossy media are also considered and the general condition on the negative refraction is derived.

1. Introduction

In 1968, Veselago [1] investigated theoretically the properties of materials with simultaneously $\varepsilon<0$ and $\mu<0$, and showed that certain uncommon phenomena occur such as a negative phase velocity, a reversed doppler shift and a left-handed relation of the vectors $\vec{E}$, $\vec{H}$ and $\vec{k}$ (hence the term left-handed media). He also concluded that the refractive indexes of such media are negative, causing a negative angle of refraction between normal and left-handed media. In 2000, Pendry [2] then showed that a planar lens made by a medium with $\varepsilon=-1$ and $\mu=-1$ can focus light on an area smaller than λ^2, which exceeds the resolution limit of ordinary lenses.

However, naturally occurring materials with simultaneously negative ε and μ have never been reported. So many researchers have employed man-made structure materials or metamaterials in order to obtain this property. For example, Pendry et al. [3,4] designed metamaterials composed of a periodic array of thin metallic wires and split ring resonators (SRRs), which provide a negative permittivity and a negative permeability, respectively. Smith et al. [5] used a certain combination of thin wires and SRRs to fabricate the first negative index metamaterial which exhibited a negative refraction at microwave frequencies. For the infrared regime, Wheeler et al. [6] designed a negative index metamaterial based upon a periodic array of polaritonic spheres coated by a drude model semiconductor. Yannopapas [7] investigated the combination of random distributed polaritonic and drude semiconductor spheres, which provide a negative index at infrared frequencies. For optical frequencies, Shalaev et al. [8] reported a nanostructured metamaterial consisting of a periodic array of pairs of parallel gold nanorods.

In isotropic lossless media, a negative phase velocity (a phase velocity which has a negative projection along the average Poynting vector) implies a negative refraction and vice versa. Hence, this property has been used as a criterion for a lossy isotropic material to possess a negative refractive index. The general condition for the negative phase velocity of a uniform plane wave in isotropic lossy media is formally proposed by McCall et al. [9], including $\mathrm{Re}\{\varepsilon\}<0$ and $\mathrm{Re}\{\mu\}<0$ as a special case. A simpler equivalent condition has been presented by Depine and Lakhtakia [10] and its applicability to active media has been discussed in Refs. [11,12]. However, in the problems of refraction concerning with lossy media, the transmitted waves are nonuniform [13] and their negative phase velocities do not always yield the negative refraction [14]. So in order to obtain a general criterion for the negative refraction in a certain medium, we should consider a refraction of the nonuniform transmitted wave rather than the phase velocity of a uniform plane wave.

Because most negative index metamaterials fabricated so far are anisotropic [5,8,9], there are also many theoretical works concerning with negative refractions in anisotropic media. For example, Mckay and Lakhtakia [14] studied the negative phase

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velocity, negative refraction and counterposition in a bianisotropic medium and showed that the positive phase velocity and the negative refraction can coexist. Furthermore, Lakhtakia and McCall [15] had previously derived a condition for counterposition phenomenon in a linear, homogeneous, dielectric–magnetic uniaxial medium for plane waves propagating from a plane interface. In related work, Zhang et al. [16] showed the total positive and negative refraction in a twin uniaxial YVO_4 bicrystal where the refraction was observed experimentally with only a very small power loss. Yonghua et al. [17] used equi-frequency contours of dispersion relation to investigate the negative refraction between isotropic–uniaxial and uniaxial–uniaxial lossless materials. Their investigation of YVO_4 crystal with the internal wavelength of 0.63 μm showed that the negative refractions can occur within a narrow range of incident angle about 10°. Ding et al. [18] presented the conditions for the negative phase velocity and the anomalous refraction in a biaxial anisotropic lossless medium, whilst Lui and Gao [19] further generalized the work of Ding et al. by covering oblique orientations of the principal axes.

In this work, we focus on orthorhombic dielectric–magnetic lossy media characterized by the relative permittivity and the relative permeability tensor $\overline{\overline{\varepsilon}}_r$ and $\overline{\overline{\mu}}_r$, respectively. In a principal axis coordinate system, these two parameters are assumed to be simultaneously diagonalizable and take the form

$$\overline{\overline{\varepsilon}}_r=\overline{\overline{\varepsilon}}/\varepsilon_0=\begin{pmatrix}\varepsilon_{rx}&0&0\\0&\varepsilon_{ry}&0\\0&0&\varepsilon_{rz}\end{pmatrix}\quad\text{and}\quad\overline{\overline{\mu}}_r=\overline{\overline{\mu}}/\mu_0=\begin{pmatrix}\mu_{rx}&0&0\\0&\mu_{ry}&0\\0&0&\mu_{rz}\end{pmatrix},\qquad(1)$$

where ε_0 and μ_0 are the permittivity and the permeability of free space, respectively. Losses in such materials are taken into account by letting the tensorial components to be complex. We also assume that the tensorial components lie on the upper half of the complex plane

$$\text{Im}\{\varepsilon_{ri}\}>0\quad\text{and}\quad\text{Im}\{\mu_{ri}\}>0,\quad i=x,y,z.\qquad(2)$$

This condition ensure that the electromagnetic energy will only be absorbed but not emitted by the medium, which was also employed in Ref. [9] for isotropic case.

We consider the TE plane wave propagations with the electric field aligned parallel to one of the principal axes. Our main objective is divided into two parts. First, we investigate the propagation of a uniform plane wave in the orthorhombic dielectric–magnetic medium and obtain the general material condition for the negative phase velocity. Second, we consider the refraction between the orthorhombic dielectric–magnetic medium and the free space and derive the general condition for the negative angle of refraction. In both cases, the numerical calculations are also performed in order to show the consistency with the theoretical results.

2. Negative phase velocity for uniform plane waves

Let us consider a uniform TE plane wave, with a z-polarized electric field, traveling parallel to the xy-plane in an orthorhombic dielectric–magnetic lossy medium (Fig. 1). The x, y, and z axes are chosen to coincide with the medium's principal axes. For this wave, we have

$$\vec{E}=\hat{z}E_0e^{j(k\hat{\rho}\cdot\vec{r}-\omega t)},\qquad(3)$$

$$\vec{H}=\overline{\overline{\mu}}_r^{-1}\cdot\left(\frac{k\hat{\rho}\times\vec{E}}{\omega\mu_0}\right)=\frac{kE_0}{\omega\mu_0}\left(\frac{\sin\phi}{\mu_{rx}}\hat{x}-\frac{\cos\phi}{\mu_{ry}}\hat{y}\right)e^{j(k\hat{\rho}\cdot\vec{r}-\omega t)},\qquad(4)$$

where the unit vector $\hat{\rho}$ lies wholly in the xy-plane, $k\hat{\rho}$ is the complex wave vector and $\overline{\overline{\mu}}_r^{-1}$ is the inverse of the relative permeability tensor. From Maxwell's equations, the dispersion relation is obtained as follows

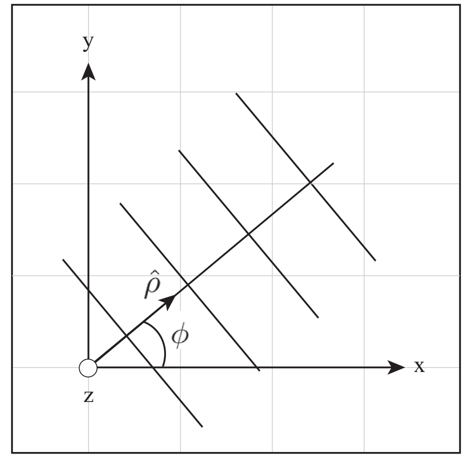

Fig. 1. A uniform plane wave propagates in the orthorhombic dielectric–magnetic medium. The electric field polarization is along the z-axis.

$$k^2=\frac{\omega^2}{c^2}\left(\frac{\varepsilon_{rz}\mu_{rx}\mu_{ry}}{\mu_{rx}\cos^2\phi+\mu_{ry}\sin^2\phi}\right).\qquad(5)$$

For simplicity of our analysis, we write Eq. (5) in the simple form:

$$k^2=\frac{\omega^2}{c^2}\varepsilon_{rz}\mu_{re},\qquad(6)$$

where

$$\mu_{re}=\frac{\mu_{rx}\mu_{ry}}{\mu_{rx}\cos^2\phi+\mu_{ry}\sin^2\phi}.\qquad(7)$$

It can easily be shown that μ_{re} also lies on the upper half of the complex plane by considering its inverse:

$$\frac{1}{\mu_{re}}=\frac{\cos^2\phi}{\mu_{ry}}+\frac{\sin^2\phi}{\mu_{rx}}.\qquad(8)$$

Since μ_{rx} and μ_{ry} are on the upper half plane, each term on the right and their sum must lie on the lower half plane. So the inverse of the sum must lie on the upper half plane.

The average Poynting vector for this wave is given by

$$\vec{S}=\frac{1}{2}\text{Re}\left\{\vec{E}\times\vec{H}^*\right\}=\frac{|E_0|^2}{2\omega\mu_0}\left[\hat{x}\,\text{Re}\left\{\frac{k\cos\phi}{\mu_{ry}}\right\}+\hat{y}\,\text{Re}\left\{\frac{k\sin\phi}{\mu_{rx}}\right\}\right]\times\exp\{-2\,\text{Im}\{k\hat{\rho}\}\cdot\vec{r}\},\qquad(9)$$

and

$$\vec{S}\cdot\hat{\rho}=\frac{|E_0|^2}{2\omega\mu_0}\text{Re}\left\{\frac{k}{\mu_{re}}\right\}\exp\left\{-2\,\text{Im}\{k\hat{\rho}\}\cdot\vec{r}\right\}.\qquad(10)$$

This indicates that some information on the alignment of $\vec{S}$ relative to $\hat{\rho}$ can be determined by considering $\text{Re}\{k/\mu_{re}\}$. Rewrite ε_{rz} and μ_{re} in polar forms as

$$\varepsilon_{rz}=|\varepsilon_{rz}|e^{i\phi_\varepsilon},\quad 0\le\phi_\varepsilon\le\pi,\qquad(11)$$

$$\mu_{re}=|\mu_{re}|e^{i\phi_\mu},\quad 0\le\phi_\mu\le\pi.\qquad(12)$$

Substituting Eqs. (11) and (12) into Eq. (6) and solving for k, we get

$$k_\pm=\pm\frac{\omega}{c}\sqrt{|\varepsilon_{rz}||\mu_{re}|}e^{i(\phi_\varepsilon+\phi_\mu)/2},\quad 0\le(\phi_\varepsilon+\phi_\mu)/2\le\pi,\qquad(13)$$

$$\frac{k_\pm}{\mu_{re}}=\pm\frac{\omega}{c}\sqrt{\frac{|\varepsilon_{rz}|}{|\mu_{re}|}}e^{i(\phi_\varepsilon-\phi_\mu)/2},\quad -\pi/2\le(\phi_\varepsilon-\phi_\mu)/2\le\pi/2,\qquad(14)$$

where k_+ and k_- lie on the upper-half and the lower-half of the complex plane, respectively. Using Eq. (14) in Eq. (10), it can be