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High-fidelity population transfer in a Josephson three-level atom with optimized level anharmonicity

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ABSTRACT

We propose a theoretical scheme to enhance the fidelity of population transfer in a Josephson three-level atom by optimizing the level anharmonicity. Without the leakage effect, the ideal population transfer can be performed via Raman adiabatic passage. In the general case, we consider the dependence of transfer fidelity on the leakage error, and then present an effective way to implement the high-fidelity population transfer using the optimized level structure. The scheme could offer a potential route towards the robust population transfer with artificial Josephson atoms experimentally.

1. Introduction

Josephson quantum circuits acting as artificial atoms possess many distinctive advantages to explore a variety of physical phenomena and laws [1–5]. Compared with a two-level atom, the three-level one has more degrees of freedom that can be controlled precisely [6]. Through adjusting external parameters, quantum coherent operations on the Josephson three-level atoms (TLAs) have been studied both experimentally and theoretically [7–11]. As a crucial issue in quantum state engineering, population transfer has attracted attention increasingly during the past years [12–16], and some novel strategies have been put forward to transfer populations with the TLAs [17–21]. Very recently, adopting the method of stimulated Raman adiabatic passage (STIRAP) [22,23], Falci et al. analyzed a Λ-type system in Cooper-pair box (CPB) circuit to study the population transfer [24].

Generally, owing to the effects of the external environment and intrinsic property [25,26], the high-fidelity population transfer with the superconducting circuits can be achieved hardly. How to enhance the fidelity is thus highly desirable to implement the perfect quantum manipulations. Utilizing the optimal control techniques as much as possible is a robust approach to improve the population transfer [27,28]. Alternatively, during the dynamical evolutions of quantum states, the intrinsic leakage caused by the external fields may be a noteworthy error source [29–31], which is associated with the level anharmonicity closely [32]. Thus, it is necessary to optimize the level anharmonicity to enhance the transfer fidelity.

In this paper, we theoretically present a feasible scheme to enhance the fidelity of population transfer in a CPB circuit by optimizing the level anharmonicity. Without the effect of leakage error, the Λ-type interaction can be obtained between the chosen TLA and the microwave pulses, by which we realize the ideal population transfer via the STIRAP. Due to the impact of leakage effect, the reduction of transfer fidelity is determined greatly by the level harmonicity. By choosing the optimized level anharmonicity, the quantum leakage can be suppressed effectively, and thereby the high-fidelity population transfer can be implemented with the available parameters. So, the proposal provides a promising approach to improve the population transfer by optimizing the intrinsic level structure.

The paper is organized as follows. In Section 2, we address an artificial TLA of a CPB circuit. In Section 3, the ideal population transfer is shown via the STIRAP. We present the high-fidelity population transfer with the optimized level anharmonicity in Section 4. Finally, discussion and conclusion are drawn in Section 5.

2. A TLA of a CPB circuit

As shown in Fig. 1(a), the CPB device under consideration includes a box with excess number n of Cooper-pairs. The box is connected to a segment of a superconducting loop via two Josephson junctions (with the identical Josephson couplings E_J and capacitances C_J). By a magnetic flux Φ_e that threads the loop, the effective Josephson energy can be tuned externally. The voltage sources V_d and $\tilde{V}_k$ are applied to the box through a gate capacitance C_g, in which the static voltage V_d modulates the system levels by offsetting gate charges, and ac ones $\tilde{V}_k$ $(k=1, 2)$ behave as classical microwaves to induce the level transitions

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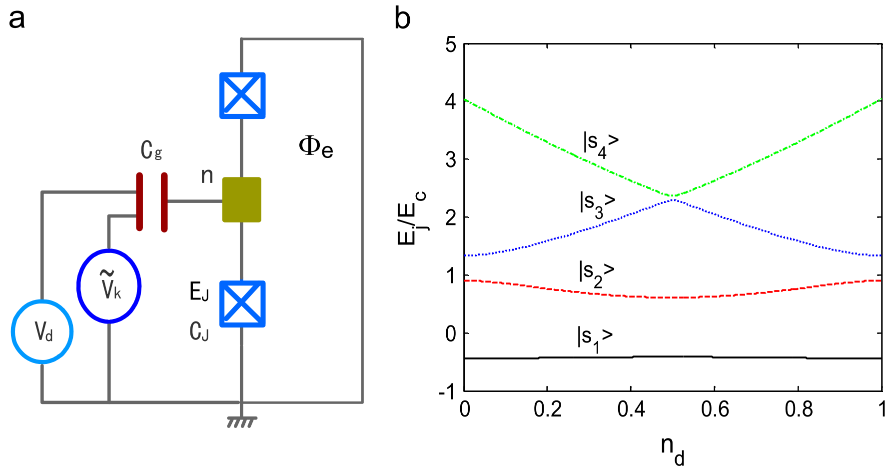

Fig. 1. (a) Schematic diagram of the considered CPB circuit and (b) the first four eigenlevels E_j of the static CPB versus n_d for the selected $\beta=1.1$, energies are given in units of E_c.

[10,33], as mentioned below. In the charge-phase regime [34], the system characteristic parameters satisfy $\Delta \gg E_c \sim E_J \gg k_B T$, where the energy gap Δ is large enough to prohibit the quasiparticle tunneling, E_c denotes the charging energy, having the same order of E_J, and $k_B T$ is the thermal excitation.

In the absence of the external drivings $\tilde{V}_k$, the static Hamiltonian of the CPB is described by $H_0 = E_c(n-n_d)^2 - \overline{E}_J \cos\theta$. The charging energy scale is $E_c = 2e^2/C_t$, with $C_t = (C_g + 2C_J)$ being the total capacitance of the box, and $n_d = C_g V_d/2e$ stands for the polarized gate charges. The tunable Josephson energy reads $\overline{E}_J = 2E_J \cos(\varphi/2)$, in which $\varphi = 2\pi\Phi_e/\Phi_0$ represents the total phase difference, and $\Phi_0 = h/2e$ indicates the flux quantum. The average phase difference θ of the two junctions is canonically conjugate to n, namely, $[\theta, n] = i$. Within the basis of Cooper-pair number state $\{|n\rangle\}$, the above Hamiltonian becomes

$$H_0 = \sum_n [E_c(n-n_d)^2 |n\rangle\langle n| - \frac{\overline{E}_J}{2}(|n\rangle\langle n+1| + H.c.)]. \tag{1}$$

Given that $\overline{E}_J$ and E_c meet the general condition of $\overline{E}_J = \beta E_c$, where β acts as the system characteristic variable and is relevant to the eigenlevels, the charging energy is $E_c = 14.18$ GHz [34]. According to Eq. (1), the first four levels E_j (j=1, 2, 3 and 4) of H_0 with the chosen $\beta = 1.1$ are given in Fig. 1(b), which are dependent on n_d. We select the lowest three levels from E_j. It is clear that the third level and the fourth one are close to each other at $n_d = 0.5$. To obtain an effective TLA, we deal with the first three levels at bias point $n_d = 0.3$. The corresponding level states $|s_r\rangle$ can be expanded in terms of Cooper-pair states $|n\rangle$, i.e., $|s_r\rangle = \sum_n c_{rn}|n\rangle$, with c_{rn} being superposition coefficients, r=1, 2 and 3.

3. Ideal population transfer via STIRAP

For the selected three-level structure, as depicted in Fig. 2(a), two microwave pulses $\tilde{V}_k$ are applied to resonantly drive the level transitions between $|s_k\rangle$ and $|s_3\rangle$, k=1, 2. The ac gate voltages have the forms $\tilde{V}_k = V_k(t)\cos(\omega_k t)$, where V_k are small and time-dependent amplitudes, and ω_k are ac frequencies that are resonantly matched with the transition frequencies $\omega_{3k} = (E_3 - E_k)/\hbar$. Since the gate pulses are diagonally coupled to the charge states, the interaction Hamiltonians between the microwave fields and the CPB system are expressed as [35]

$$H_{ks} = -2E_c \tilde{n}_k \sum_n (n-n_d)|n\rangle\langle n|, \tag{2}$$

where $\tilde{n}_k = n_k(t)\cos(\omega_k t)$, with $n_k(t) = C_g V_k(t)/2e$ being the reduced microwave amplitudes. The fast oscillating terms such as $\tilde{n}_k^2$ have been ignored in the rotating wave approximation (RWA). Under the classical microwave radiations, the matrix elements describing the transitions between $|s_k\rangle$ and $|s_3\rangle$ are

$$t_{k3} = \langle s_k|H_{ks}|s_3\rangle = -2E_c\tilde{n}_k O_{k3}, \tag{3}$$

where $O_{k3} = \sum_n (n-n_d) c_{kn}^* c_{3n}$ are the wavefunction overlap between $|s_k\rangle$ and $|s_3\rangle$ [10]. As a result, the relevant Rabi frequencies are $\Omega_{k3} = n_k E_c |O_{k3}|/\hbar$ in the RWA.

In the interaction picture, the effective Hamiltonian of the Λ-type interaction between the TLA and the microwave pulses is given by

$$H_{eff}^{(1)} = \hbar(\Omega_{13}|s_1\rangle\langle s_3| + \Omega_{23}|s_2\rangle\langle s_3|) + H.c., \tag{4}$$

where Ω_{13} and Ω_{23} are dependent on the tunable microwave amplitudes $n_1(t)$ and $n_2(t)$, respectively. The dynamical evolution of an arbitrary state ψ is governed by the Schrödinger equation $i\hbar\dot{\psi} = H_{eff}^{(1)}\psi$, where $\psi = \sum_r c_r|s_r\rangle$, with c_r being the superposition coefficients. Within the basis of $\{|s_r\rangle\}$, the above equation can be rewritten as the matrix form

$$i\frac{d}{dt}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \Omega_{13} \\ 0 & 0 & \Omega_{23} \\ \Omega_{13}^* & \Omega_{23}^* & 0 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}. \tag{5}$$

Now consider n_1 and n_2 as Gaussian functions of time [17], $n_1 = 0.06e^{-(t-\tau_1)^2/\tau^2}$ and $n_2 = 0.03e^{-(t-\tau_2)^2/\tau^2}$, where 0.06 and 0.03 are the maximum amplitudes of pulses, $\tau_1 = 115$ ns, $\tau_2 = 75$ ns, and $\tau = 35$ ns are the pulse-related parameters, respectively. At the working point $n_d = 0.3$, we numerically obtain $O_{13} = -0.132$ and $O_{23} = 0.387$. Thus the Rabi frequencies Ω_{k3} are demonstrated in Fig. 2(b). Assume that the system is initially in $|s_1\rangle$, we get the state evolutions by solving Eq. (5). As plotted in Fig. 2(c), the population is transferred from the initial $|s_1\rangle$ to the target state $|s_2\rangle$ through the intermediate one $|s_3\rangle$ nearly. During the transfer process, the applied pulses are required to perform the adiabatic operations. With the effective Rabi frequencies $\Omega_{13} = 0.11e^{-(t-\tau_1)^2/\tau^2}$ GHz and $\Omega_{23} = 0.16e^{-(t-\tau_2)^2/\tau^2}$ GHz, we have $\int_0^{t_f}\Omega_{13}\,dt = 6.97$ and $\int_0^{t_f}\Omega_{23}\,dt = 10.2$ ($t_f = 200$ ns), which approximately meet the adiabatic conditions. Meanwhile, a sequence of two partially overlapping pulses is applied in counterintuitive order: first $\tilde{V}_2$ and then $\tilde{V}_1$, the relevant Rabi frequencies Ω_{13} and Ω_{23} traverse the closed-loop in real-time space. Therefore, the coherent quantum operation is just referred to as the STIRAP [22,23].

The present pulse parameters are chosen for two reasons. First, to ignore the pulse-induced level fluctuations effectively, the maximum pulse amplitudes (0.06 and 0.03), representing the pulses induced