Optics Communications 316 (2014) 132–139

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Statistical properties of a partially coherent radially polarized beam propagating through an astigmatic optical system

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ARTICLE INFO

Article history:
Received 2 October 2013
Received in revised form
29 November 2013
Accepted 1 December 2013
Available online 14 December 2013

Keywords:
Partially coherence
Radially polarized beam
Astigmatism
Propagation

ABSTRACT

In a recent publication (Wang et al., Appl. Phys. Lett. 100 (2012) 051108), we reported experimental generation of a partially coherent radially polarized (RP) beam. In this paper, based on the unified theory of coherence and polarization, we derive the analytical expressions for the elements of the cross-spectral density matrix of a partially coherent RP beam propagating through an astigmatic optical system. As an application example, we study numerically the effect of astigmatism of the optical system on the focusing properties of a partially coherent RP beam. Our results show that the astigmatism of the optical system can be used for modulating the statistical properties of a focused partially coherent RP beam, which will be useful for beam shaping and particle trapping.

1. Introduction

Radially polarized beam is a typical kind of cylindrical vector beam, which has been studied extensively both in theory and experiment in the past decade due to its unique tight focusing properties by a high numerical aperture objective lens and important applications in various fields, such as microscopy, lithography, hologram, electron acceleration, proton acceleration, particle trapping, material processing, optical data storage, high-resolution metrology, super-resolution imaging, plasmonic focusing, free-space optical communications, and laser machining [1–14]. All the above-mentioned literatures have been confined to coherent RP beams.

Recently, more and more attention is being paid to partially coherent vector beams [15–44]. Partially coherent vector beam with spatially uniform state of polarization (e.g., elliptically polarized beam and circularly polarized beam) usually is called stochastic (i.e., random) electromagnetic beam or partially coherent and partially polarized beam [15,16]. Since the unified theory of coherence and polarization was proposed by Wolf [15], stochastic electromagnetic beam has been studied extensively both in theory and experiment [17–33]. Propagation properties of a stochastic electromagnetic beam through an astigmatic optical system have been studied in detail [31–33].

Partially coherent vector beam with spatially non-uniform state of polarization is called partially coherent cylindrical vector beam which was introduced recently [34,35]. As a special case of partially coherent cylindrical vector beam, partially coherent RP beam was introduced in theory [36–39] and generated in experiment [40] recently. Properties of a partially coherent radially polarized beam focused by a stigmatic optical system of arbitrary numerical aperture and Fresnel number were reported in Ref. [39]. Coherence and polarization properties of a partially coherent RP beam on propagation were measured in Ref. [41]. More recently, we reported experimental study of the scintillation properties of a partially coherent RP beam propagating through thermally induced turbulence and it was revealed that a partially coherent RP beam has advantage over a linearly polarized partially coherent Gaussian beam for reducing turbulence-induced scintillation, which will be useful in free-space optical communications [42]. To our knowledge no results have been reported up until now on the propagation of a partially coherent RP beam through an astigmatic optical system. In this paper, our aim is to study the propagation of a partially coherent RP beam through an astigmatic optical system, and investigate the effect of astigmatism of the optical system on the focusing properties of a partially coherent RP beam. Some interesting and useful results are found.

2. Theory

Based on the unified theory of coherence and polarization, the second-order statistical properties of a partially coherent vector beam can be characterized by the following 2×2 cross-spectral density (CSD) matrix [15]

$$\overleftrightarrow{W}(x_1,y_1,x_2,y_2,\omega)=\begin{pmatrix} W_{xx}(x_1,y_1,x_2,y_2,\omega) & W_{xy}(x_1,y_1,x_2,y_2,\omega) \\ W_{yx}(x_1,y_1,x_2,y_2,\omega) & W_{yy}(x_1,y_1,x_2,y_2,\omega) \end{pmatrix}, \quad (1)$$

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http://dx.doi.org/10.1016/j.optcom.2013.12.002

where (x_1, y_1) and (x_2, y_2) denotes two arbitrary position vectors in the source plane, and

$$W_{\alpha\beta}(x_1, y_1, x_2, y_2, \omega) = \langle E_\alpha(x_1, y_1, \omega) E^*_\beta(x_2, y_2, \omega)\rangle, (\alpha = x, y; \beta = x, y) \tag{2}$$

Here E_x and E_y denote the components of the random electric vector, with respect to two mutually orthogonal, x and y directions, perpendicular to the z-axis. The asterisk denotes the complex conjugate and the angular brackets denote the ensemble average.

The elements of the CSD matrix of a partially coherent RP beam in the source plane are expressed as follows [40–42]

$$W_{xx}(x_1, y_1, x_2, y_2, \omega) = \frac{x_1 x_2}{\omega_0^2}\exp\left[-\frac{x_1^2+x_2^2}{\omega_0^2} - \frac{y_1^2+y_2^2}{\omega_0^2} - \frac{(x_1-x_2)^2}{2\delta_0^2} - \frac{(y_1-y_2)^2}{2\delta_0^2}\right], \tag{3}$$

$$W_{yy}(x_1, y_1, x_2, y_2, \omega) = \frac{y_1 y_2}{\omega_0^2}\exp\left[-\frac{x_1^2+x_2^2}{\omega_0^2} - \frac{y_1^2+y_2^2}{\omega_0^2} - \frac{(x_1-x_2)^2}{2\delta_0^2} - \frac{(y_1-y_2)^2}{2\delta_0^2}\right], \tag{4}$$

$$W_{xy}(x_1, y_1, x_2, y_2, \omega) = \frac{x_1 y_2}{\omega_0^2}\exp\left[-\frac{x_1^2+x_2^2}{\omega_0^2} - \frac{y_1^2+y_2^2}{\omega_0^2} - \frac{(x_1-x_2)^2}{2\delta_0^2} - \frac{(y_1-y_2)^2}{2\delta_0^2}\right], \tag{5}$$

$$W_{yx}(x_1, y_1, x_2, y_2, \omega) = W^*_{xy}(x_2, y_2, x_1, y_1, \omega),) \tag{6}$$

where ω_0 and δ_0 represent the transverse beam size and the transverse spatial coherence width, respectively.

Within the validity of the paraxial approximation, the propagation of the elements of the CSD matrix of a partially coherent vector beam through an astigmatic ABCD optical system can be studied with the help of the following generalized Collins formula [31,32,43]

$$\begin{aligned} W_{\alpha\beta}(u_1, v_1, u_2, v_2, \omega) &= \left(\frac{1}{\lambda|B|}\right)^2 \int\int\int\int W_{\alpha\beta}(x_1, y_1, x_2, y_2, \omega)\exp\left[ikC_6(x_1^2-x_2^2) - ikC_6(y_1^2-y_2^2)\right] \\ &\times\exp\left[-\frac{ik}{2B}(Ax_1{}^2 - 2x_1u_1 + Du_1^2) - \frac{ik}{2B}(Ay_1{}^2 - 2y_1v_1 + Dv_1^2)\right] \\ &\times\exp\left[\frac{ik}{2B}(Ax_2^2 - 2x_2u_2 + Du_2^2) + \frac{ik}{2B}(Ay_2^2 - 2y_2v_2 + Dy_2^2)\right]dx_1dx_2dy_1dy_2, \end{aligned} \tag{7}$$

where $k = 2\pi/\lambda$ is the wave number with λ being the wavelength. A–D are the elements of the transfer matrix of the optical system. The astigmatism in the ABCD optical system is taken into consideration through the term $\exp\left[ikC_6(x_1^2-x_2^2) - ikC_6(y_1^2-y_2^2)\right]$ with C_6 being the astigmatic coefficient [43].

Substituting Eqs. (3)–(6) into Eq. (7), we obtain (after tedious integration and operation) the following expressions for the elements of the CSD matrix of the partially coherent RP beam after passing through an astigmatic ABCD optical system

$$\begin{aligned} W_{xx}(u_1, v_1, u_2, v_2, \omega) &= \frac{k^2}{16B^2\omega_0^2(\alpha\beta)^{\frac{3}{2}}\sqrt{\gamma\eta}}\exp\left[-\frac{ikD}{2B}(u_1^2+v_1^2) + \frac{ikD}{2B}(u_2^2+v_2^2) - \frac{k^2u_1^2\gamma + k^2v_1^2\alpha}{4\alpha\gamma B^2}\right] \\ &\times\exp\left[\frac{-k^2u_1^2 + 4k^2u_1u_2\alpha\delta_0^2 - 4k^2u_2^2\alpha^2\delta_0^4}{16\alpha^2\beta B^2\delta_0^4} + \frac{-k^2v_1^2 + 4k^2v_1v_2\gamma\delta_0^2 - 4k^2v_2^2\gamma^2\delta_0^4}{16\gamma^2\eta B^2\delta_0^4}\right] \\ &\times\left(\frac{-k^2u_1^2 + 2k^2u_1u_2\alpha\delta_0^2}{2\alpha B^2\delta_0^2} + \frac{8\alpha^2\beta B^2\delta_0^4 - k^2u_1^2 + 4k^2u_1u_2\alpha\delta_0^2 - 4k^2u_2^2\alpha^2\delta_0^4}{8\alpha^2\beta B^2\delta_0^6}\right), \end{aligned} \tag{8}$$

$$\begin{aligned} W_{yy}(u_1, v_1, u_2, v_2, \omega) &= \frac{k^2}{16B^2\omega_0^2(\gamma\eta)^{\frac{3}{2}}\sqrt{\alpha\beta}}\exp\left[-\frac{ikD}{2B}(u_1^2+v_1^2) + \frac{ikD}{2B}(u_2^2+v_2^2) - \frac{k^2v_1^2\alpha + k^2u_1^2\gamma}{4\gamma\alpha B^2}\right] \\ &\times\exp\left[\frac{-k^2u_1^2 + 4k^2u_1u_2\alpha\delta_0^2 - 4k^2u_2^2\alpha^2\delta_0^4}{16\alpha^2\beta B^2\delta_0^4} + \frac{-k^2v_1^2 + 4k^2v_1v_2\gamma\delta_0^2 - 4k^2v_2^2\gamma^2\delta_0^4}{16\gamma^2\eta B^2\delta_0^4}\right] \\ &\times\left[\frac{-k^2v_1^2 + 2k^2v_1v_2\gamma\delta_0^2}{2\gamma B^2\delta_0^2} + \frac{8\gamma^2\eta B^2\delta_0^4 - k^2v_1^2 + 4k^2v_1v_2\gamma\delta_0^2 - 4k^2v_2^2\gamma^2\delta_0^4}{8\gamma^2\eta B^2\delta_0^6}\right], \end{aligned} \tag{9}$$

$$\begin{aligned} W_{xy}(u_1, v_1, u_2, v_2, \omega) &= \frac{k^2}{16B^2\omega_0^2(\alpha\eta)^{\frac{3}{2}}\sqrt{\gamma\beta}}\left(\frac{4iku_1\alpha\beta\delta_0^4 + iku_1 - 2iku_2\alpha\delta_0^2}{4\alpha\beta B\delta_0^4}\right)\left(\frac{ikv_1 - 2ikv_2\gamma\delta_0^2}{2\gamma B\delta_0^2}\right) \\ &\times\exp\left[\frac{-k^2u_1^2 + 4k^2u_1u_2\alpha\delta_0^2 - 4k^2u_2^2\alpha^2\delta_0^4}{16\alpha^2\beta B^2\delta_0^4} + \frac{-k^2v_1^2 + 4k^2v_1v_2\gamma\delta_0^2 - 4k^2v_2^2\gamma^2\delta_0^4}{16\gamma^2\eta B^2\delta_0^4}\right] \\ &\times\exp\left[-\frac{ikD}{2B}(u_1^2+v_1^2) + \frac{ikD}{2B}(u_2^2+v_2^2) - \frac{k^2u_1^2\gamma + k^2v_1^2\alpha}{4\alpha\gamma B^2}\right], \end{aligned} \tag{10}$$

$$W_{yx}(u_1, v_1, u_2, v_2, \omega) = W^*_{xy}(u_2, v_2, u_1, v_1, \omega), \tag{11}$$

where (u_1, v_1) and (u_2, v_2) are two arbitrary position vectors in the output plane and

$$\alpha = \frac{1}{\omega_0^2} + \frac{1}{2\delta_0^2} - ikC_6 + \frac{ikA}{2B}, \quad \beta = \frac{1}{\omega_0^2} + \frac{1}{2\delta_0^2} + ikC_6 - \frac{ikA}{2B} - \frac{1}{4\alpha\delta_0^4}, \tag{12}$$

$$\gamma = \frac{1}{\omega_0^2} + \frac{1}{2\delta_0^2} + ikC_6 + \frac{ikA}{2B}, \quad \eta = \frac{1}{\omega_0^2} + \frac{1}{2\delta_0^2} - ikC_6 - \frac{ikA}{2B} - \frac{1}{4\gamma\delta_0^4} \tag{13}$$

Eqs. (8)–(11) are the main analytical results of this paper.

The spectral intensity of the partially coherent RP beam at point (u, v) is given by Ref. [15]

$$\langle I(u, v, \omega)\rangle = \mathrm{Tr}\overleftrightarrow{W}(u, v, u, v, \omega) = W_{xx}(u, v, u, v, \omega) + W_{yy}(u, v, u, v, \omega), \tag{14}$$

The degree of polarization of the partially coherent RP beam at point (u, v) is defined as Ref. [15]

$$P(u, v, \omega) = \sqrt{1 - \frac{4\mathrm{Det}\overleftrightarrow{W}(u, v, u, v, \omega)}{[\mathrm{Tr}\overleftrightarrow{W}(u, v, u, v, \omega)]^2}}, \tag{15}$$

where Det denotes the determinant of the CSD matrix.

The spectral degree of coherence of the partially coherent RP beam at a pair of transverse points with position vectors (u_1, v_1) and (u_2, v_2) is defined as Ref. [15]

$$\mu(u_1, v_1, u_2, v_2, \omega) = \frac{\mathrm{Tr}\overleftrightarrow{W}(u_1, v_1, u_2, v_2, \omega)}{\sqrt{\mathrm{Tr}\overleftrightarrow{W}(u_1, v_1, u_2, v_2, \omega)\mathrm{Tr}\overleftrightarrow{W}(u_1, v_1, u_2, v_2, \omega)}} \tag{16}$$