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Extracting inter-particle forces in opaque granular materials: Beyond photoelasticity

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ABSTRACT

This paper presents the first example of inter-particle force inference in real granular materials using an improved version of the methodology known as the Granular Element Method (GEM). GEM combines experimental imaging techniques with equations governing particle behavior to allow force inference in cohesionless materials with grains of arbitrary shape, texture, and opacity. This novel capability serves as a useful tool for experimentally characterizing granular materials, and provides a new means for investigating force networks. In addition to an experimental example, this paper presents a precise mathematical formulation of the inverse problem involving the governing equations and illustrates solution strategies.

1. Introduction

Granular materials are ubiquitous in nature and technology: soils, foods, industrial products, and many other natural and engineered materials are granular (Jaeger et al., 1996). Although these materials are governed by seemingly simple laws on the particle scale, their observed mesoscopic and macroscopic behavior is complex and not fully explained by any unified theory. A primary goal in the study of granular materials has therefore been to understand their behavior from a micromechanical standpoint using only simple mechanical laws and observations of particle level phenomena.

Significant progress has been made in studying microscopic features of granular materials. Inter-particle forces and force chains have received extensive attention (e.g., Cates et al., 1998; Peters et al., 2005 and references therein). Studies have examined relations between inter-particle forces and bulk properties (Rothenburg and Bathurst, 1989; Bathurst and Rothenburg, 1990). Extensive research has focused on collective statistical properties of inter-particle forces (Majmudar and Behringer, 2005; Coppersmith et al., 1996; Guo and Zhao, 2013; Liu et al., 1995; Ostojic et al., 2006; Radjai et al., 1996, 1998; Satake, 1982). Many models have emerged to simulate the observed heterogeneities of force networks (Bouchaud et al., 2001; Claudin et al., 1998; Liu et al., 1995). Modeling techniques embracing the link between inter-particle forces and macroscopic properties have also emerged and successfully reproduced many observed features of granular materials (Andrade and Tu, 2009; Andrade et al., 2011). The progress and success of these studies hinges on the ability to validate theories with experiments on real granular materials. To this end, experimental techniques for inferring inter-particle forces are essential.

Several experimental techniques have historically provided powerful methods for inferring inter-particle forces in granular materials. Photoelasticity has provided the most popular and widely used technique (Drescher and de Josselin de Jong, 1972; Howell et al., 1999). While photoelasticity has and will continue to occupy an important place in the study

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of granular materials, it has many limitations: it requires the use of birefringent grains with simple geometries, is difficult to implement in three dimensions, and it often requires knowledge of boundary forces for accurate results (Frocht, 1941a,b). Other methods proposed recently attempt to overcome some of these limitations but are also restricted to particular materials or grain shapes (see e.g., Saadatfar et al., 2012; Zhou et al., 2006). For example, Saadatfar et al. (2012), Zhou et al. (2006) are restricted to spherical particles and rely on a particular choice of contact law. As a consequence, the materials of choice have to be relatively soft, e.g., rubber or droplets. Also, the droplets in Zhou et al. (2006) are frictionless. While these methods constitute a great first step in the desired direction, there are still no experiment-based quantitative values of interparticle forces reported in the literature. The methodology presented in this paper leverages emerging experimental techniques to overcome many of these limitations: it can be applied to real, complex materials of arbitrary shape and opacity, is compatible with a number of imaging methods (including x-ray diffraction and tomography), it does not rely on any contact law, and has a simple and physically intuitive mathematical structure.

The Granular Element Method (GEM) presented in this paper is an extended form of the method originally proposed in Andrade and Avila (2012). The contribution of this paper is to present an experimental validation of GEM representing the first application of GEM to infer inter-particle forces in complex opaque materials and to present a precise mathematical formulation of the proposed inverse problem involving the governing equations for particle mechanics. This paper also provides an additional inverse problem formulation that may help to practitioners reduce solution error when experimental noise is present. The contribution of the GEM framework itself is more profound: GEM provides a general framework for inferring inter-particle forces in two or three dimensions and in any granular material, provided that the necessary data can be obtained through experiments.

The GEM methodology can be visualized in Fig. 1. Experimental imaging techniques such as high-resolution photography, 3D X-ray diffraction (Hall et al., 2011; Martins et al., 2004; Poulsen, 2004), and X-ray computed tomography (XRCT) (Alshibli and Reed, 2010; Desrues et al., 2006; Wang et al., 2007) provide rich data sets from which Digital Image Correction (DIC) (Sutton et al., 2009), level-set methods (Vlahinic et al., under review), and other techniques can extract intra-particle strain fields and material fabric (contact locations and normals). Intra-particle strains fields and material fabric are used as input into a mathematical framework which yields intra-particle stress fields using an appropriate constitutive relation and numerically solves an inverse problem using the governing equations of particle statics. The result of the inverse problem is inter-particle forces, a key component in the development of theories regarding granular behavior. Because GEM can be coupled with any experimental techniques capable of extracting strain fields and fabric, recent advances in imaging (e.g., Hall et al., 2011; Martins et al., 2004) will soon allow inference of inter-particle forces in natural materials with small grains, such as sands.

The layout of this paper is as follows. Section 2 presents an example of the methodology applied to a real experiment involving rubber particles, showcasing the applicability of the method to real materials of any texture and its adaptability to a number of experimental imaging and algorithmic data analysis techniques. Sections 3 and 4 detail the ingredients of the mathematical framework of GEM. Section 5 illustrates additional features of GEM with a numerical example. Finally, Section 6 offers concluding remarks.

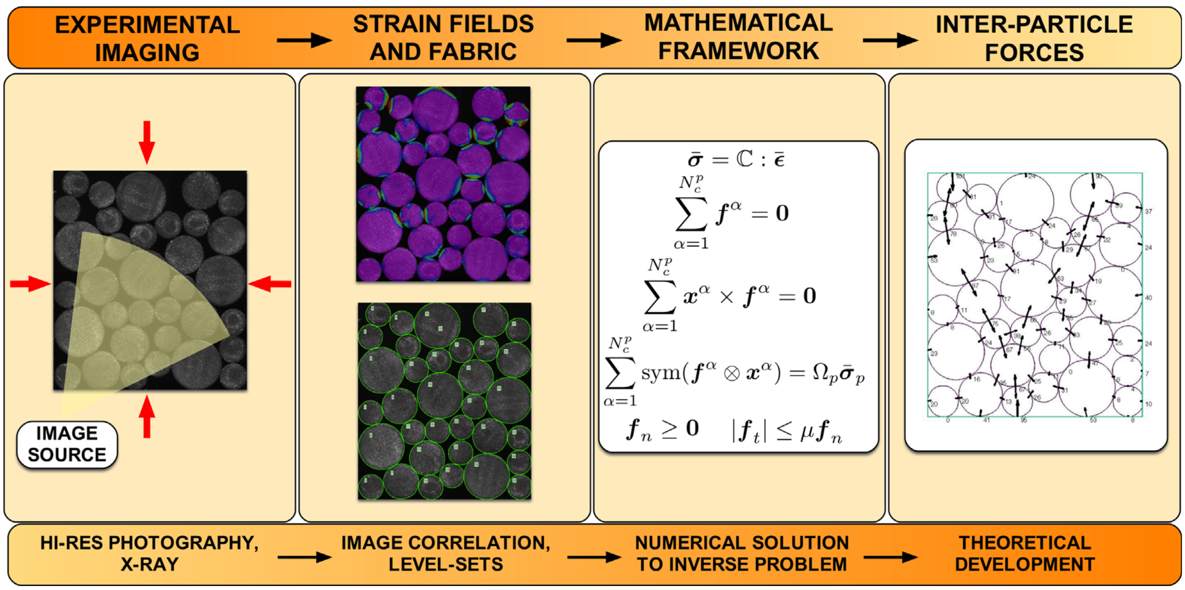

Fig. 1. The GEM methodology for inter-particle force-inference presented in this paper. Experimental imaging techniques provide rich data sets for extracting intra-particle strain fields and material fabric. These ingredients are input into a mathematical framework which yields inter-particle forces by solving an appropriate inverse problem. Variables and equations in panel 3 are described in Section 3.