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Theory of cyclic creep of concrete based on Paris law for fatigue growth of subcritical microcracks

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ABSTRACT

Recent investigations prompted by a disaster in Palau revealed that worldwide there are 69 long-span segmental prestressed-concrete box-girder bridges that suffered excessive multi-decade deflections, while many more surely exist. Although the excessive deflections were shown to be caused mainly by obsolescence of design recommendations or codes for static creep, some engineers suspect that cyclic creep might have been a significant additional cause. Many investigators explored the cyclic creep of concrete experimentally, but a rational mathematical model that would be anchored in the microstructure and would allow extrapolation to a 100-year lifetime is lacking. Here it is assumed that the cause of cyclic creep is the fatigue growth of pre-existing microcracks in hydrated cement. The resulting macroscopic strain is calculated by applying fracture mechanics to the microcracks considered as either tensile or, in the form of a crushing band, as compressive. This leads to a mathematical model for cyclic creep in compression, which is verified and calibrated by laboratory test data from the literature. The cyclic creep is shown to be proportional to the time average of stress and to the 4th power of the ratio of the stress amplitude to material strength. The power of 4 is supported by the recent finding that, on the atomistic scale, the Paris law should have the exponent of 2 and that the exponent must increase due to scale bridging. Exponent 4 implies that cyclic creep deflections are enormously sensitive to the relative amplitude of the applied cyclic stress. Calculations of the effects of cyclic creep in six segmental prestressed concrete box girders indicate that, because of self-weight dominance, the effect on deflections absolutely negligible for large spans (>150 m). For small spans (<40 m) the cyclic creep deflections are not negligible but do not matter since the static creep causes in such bridges upward deflections. However, the cyclic creep is shown to cause in bridges with medium and small spans (<80 m) a significant residual tensile strain which can produce deleterious tensile cracking at top or bottom face of the girder.

1. Introduction

A segmental prestressed-concrete box-girder bridge built in 1977 in Palau, which had the world record span of 241 m, deflected within 18 years by 1.61 m. This was 5.3-times greater than the allowable deflection. An attempt to lift the midspan

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by additional prestressing led three month later to collapse (with fatalities). Subsequent worldwide search for data (Bazant et al., 2011a) revealed that 69 large-span bridges of the same type suffered within 20–40 years long-time deflections most of which are excessive and require either replacement of the bridge or risky remedial prestressing. Although a detailed analysis (Bazant et al., 2012a, 2012b) showed that these excessive deflections can be explained by severe underestimation of multidecade (static) creep of concrete in the current design codes and standard recommendations (ACI Committee 209, 1972, 2008; FIB, 1999; Gardner, 2000; Gardner and Lockman, 2001; Bazant and Baweja, 1995, 2000), some engineers at conferences questioned whether the cyclic creep due to traffic loads may have been a significant contributing factor. Examination of this question was the motivation for the present study in which a constitutive law for cyclic creep is developed and calibrated by the available test data.

The cyclic creep of concrete, also called the fatigue creep or vibro-creep (vibropolzuchest' in Russian), is the long-time deformation produced by cyclic load in excess of the static creep. This phenomenon was experimentally detected by Féret in 1906 and was also observed by Probst in 1925, Mehmel and Heim in 1926, and Ban in 1933 (cf. Bechyně, 1959). More systematic experiments that allow quantitative characterization had to wait until the works of Kern and Mehmel (1962) and Gaede (1962). After World Word II, many researchers studied this phenomenon experimentally and proposed various approximate empirical formulas (Le Camus, 1946; L'Hermite, 1961; Mal'mejster, 1957; L'Hermite, 1961; Murdock, 1965; Gvozdev, 1966; Bennett and Muir, 1967; Nordby, 1967; Bazant, 1968a, 1968b; Batson et al., 1972; Wittmann, 1971; Whaley and Neville, 1973; Hirst and Neville, 1977; Neville and Hirst, 1978; Bazant and Panula, 1979; Garrett et al., 1979; Hsu, 1981; Brooks and Forsyth, 1986; Bazant et al., 1992; Pandolfi and Taliercio, 1998). Yet mutually contradictory formulations abound and no generally accepted theory has yet emerged.

In 1962, Gaede (1962) proposed the following formula based on his own extensive tests of cyclic compression:

$$\Delta\varepsilon_N = \frac{cN_0}{f_p}\frac{\sigma_{max}}{E_{sec}}\left(\frac{N}{N_0}\right)^r \tag{1}$$

where $\Delta\varepsilon_N =$ strain increment due to cyclic loading; $N =$ number of cycles, $N_0 = 10^5$; $f_p =$ compressive strength of prisms; c and r=empirical fitting parameters with no mechanical explanation provided; and $E_{sec} =$ instantaneous elastic modulus measured for pulsating compression. Eq. (1) was based on compression cycles from 14% to 75% of the compression strength f'_c, which is way beyond the service stress range allowed in bridge design ($<40\%$ of f'_c).

Wittmann (1971) tried to generalized his power law for a (static) creep curve, $\varepsilon(t) = at^n \sinh(b\sigma/f_c)$ in which a, b are empirical constants. The aging effect was ignored, and a hyperbolic sine function ensued from an assumption of thermally activated transitions. Considering cyclic stress $\sigma = \sigma_m + \Delta\sigma \sin \omega t$ where ω=constant and $\sigma_m, \Delta\sigma =$ the mean and amplitude of cyclic stress, he empirically generalized this power law by replacing the constant exponent n with the variable $n = n_0 + c(\Delta\sigma/f_c)^d$ where n_0, c, d are constants which he calibrated by Gaede's data.

Hirst and Neville (1977), Neville and Hirst (1978) and Whaley and Neville (1973) presented the most comprehensive and diverse experimental data. In Neville and Hirst (1978), they proposed that the cyclic creep is an inelastic deformation caused by microcracking, but made no attempt to model the microcracking per se. In view of the hardening effect under low stress cyclic creep observed in some experiments (Bennett and Muir, 1967; Hirst and Neville, 1977; Neville and Hirst, 1978; Whaley and Neville, 1973; Batson et al., 1972), they suggested that the microcracking occurs at the aggregate interfaces. Garrett and Jennings, Garrett et al. (1979), speculated that these microcracks could expose unhydrated cement to further hydration which in turn might cause further deformation. Hirst, and later also Brooks Hirst and Neville (1977); Brooks and Forsyth (1986), assumed that $\varepsilon_{\text{cyclic}} = \varepsilon_{\text{static}} A(\ln t)^B$, where ε is the creep strain and A, B are calibration parameters.

Later on, Pandolfi and Taliercio (1998) suggested a more complicated formula for cyclic creep of concrete based on numerical simulations. They emphasized two concepts: The time is only implicitly related to the number of cycles, N, i.e., the tests should be interpreted in terms of N, the number of cycles, and the loading frequency is indirectly related to the loading rate (Hsu, 1981). Damage evolution models based on failure surfaces in the stress space have also been suggested. It appears, however, that no model based on fatigue growth of individual microcracks under cyclic loading has been presented.

The phenomenological formulations treated cyclic creep in two ways: either as a deformation $\Delta\varepsilon_N$ that is additional to the static creep (Bazant, 1968a) or as an acceleration of the static creep (Bazant and Panula, 1979; Bazant et al., 1992, 2012b). Both were able to provide acceptable fit of the main data, doubtless because of their limited duration (mostly <10 days, some up to 28 days). However, extrapolations to lifetimes of up to 100 or 150 years, usually desired for large bridges, give very different predictions.

In this paper, a micro-mechanical model for cyclic creep is proposed and calibrated by tests. Although the micro-mechanics has been discussed intuitively in qualitative terms (e.g., Neville and Hirst, 1978; Garrett et al., 1979) or in terms of phenomenological damage mechanics (Gao and Hsu, 1998; Maekawa and El-Kashif, 2013), no micro-mechanics based and experimentally validated constitutive model seems to have been proposed during a century of research. This is what the present study aspires to.

2. What is the microscale mechanism?

In fatigue embrittled metals or in fine-grained ceramics, a critical safety consideration is the fatigue growth of cracks. Although such dangerous cracks can be small, they are nevertheless macrocracks because they are larger than the