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Nonlinear analysis of microstructure-dependent functionally graded piezoelectric material actuators

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ABSTRACT

In the present work, a nonlinear thermo-electro-mechanical response of functionally graded piezoelectric material (FGPM) actuators is investigated. The theoretical formulation is based on the Timoshenko beam theory with the von Kármán nonlinearity (in the form of midplane stretching), and a microstructural length scale is incorporated by means of the modified couple stress theory. A power-law distribution of thermal, electrical, and mechanical properties through beam thickness (or height) is assumed. The governing equations are derived using the principle of virtual displacements. A displacement finite element model of the theory is developed, and the resulting system of nonlinear algebraic equations is solved with the help of Newton's iteration method. Numerical results are presented for transverse deflection as a function of load parameters and out-of-plane boundary conditions. The parametric effects of microstructural length scale parameter, power-law index of the material distribution across the thickness, boundary conditions, beam geometry, and applied actuator voltage on the beam response are investigated through various numerical examples. The results reveal the existence of bifurcation (or critical states) for certain types of in-plane loads. For other load types, including out-of-plane loads, the beam undergoes a unique and stable deflection path that does not contain any critical point.

1. Introduction

Piezoelectric materials are used in various types of structures to suppress certain vibration modes, control shape, and postpone initial buckling (see Tzou, 1993; Yang, 2005, 2006). Beam-like structures are commonly used as sensors and actuators in a variety of mechanical, civil, and structural applications at various scales. Therefore, analysis of beams made of piezoelectric materials or those incorporated with piezoelectric patches is of great importance. For beams with rectangular cross-sectional areas, by applying double power-series solutions developed in thickness and width directions, Yang and his co-authors (Yang and Zhang, 1999; Yang et al., 1999; Yang, 1998) analyzed three-dimensional behavior of electro-elastic beams in which extensional and transverse motions are accounted for. Based on the conventional Euler–Bernoulli beam theory, Wang and Quek (2002, 2012) investigated the natural vibration of a beam with piezoelectric layers.

The concept of mixing two or more materials with a predetermined variation of volume fractions of the constituents to achieve certain functionality – such as thermally inertness, fracture toughness, and so on – was conceived experimentally by

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Japanese in 1960s, although such materials existed in the nature (e.g., sea shells, tree trunks, walls of arteries in human body, and other systems). Early research on FGM structures is documented in the works of Hasselman and Youngblood (1978), Yamanouchi et al. (1990), Koizumi (1993), and Reddy and his colleagues (Reddy and Chin, 1998; Praveen and Reddy, 1998), among many others. Piezoelectric FGMs are developed to eliminate stress concentration while having variable sensing and actuation properties. Reported works on functionally graded piezoelectric materials (FGPMs) under thermo-electro-mechanical loads are limited in number. Based on the two-dimensional theory of elasticity, a solution for the response of an FGPM beam with an arbitrary distribution of two materials constituents through the beam thickness is provided by Huang et al. (2007). In their study, the equilibrium and Maxwell equations are satisfied by introducing a stress function and an electrical displacement function. Distributions of the stress and electrical displacement functions are assumed to be quadratic through the thickness. Series of investigations on orthotropic FGPM beams are performed by Shi (2002), Shi and Chen (2004), and Liu and Shi (2004). In Shi (2002), analytical solutions for a cantilever beam, where density varies as a cubic polynomial across the thickness, are provided. The case of quadratic and cubic variations of Young's modulus and density of the beam across the thickness are considered by Shi and Chen (2004). By proposing a stress function and with consideration of linearly graded piezoelectric parameter through-the-thickness, Liu and Shi (2004) obtained the response of an FGPM beam.

The most comprehensive study on the geometrically linear static, dynamic, and free vibration response of FGPM beams under the action of various types of excitations has been performed by Yang and Xiang (2007). In this work, three mechanical equations and the Maxwell equation are solved simultaneously via the differential quadrature (DQ) method. Doroushi et al. (2011) reported dynamic response of FGPM beams based on the third-order shear deformation theory of Reddy (1984, 2004a) and the finite element method.

Among the works reported on thermo-electro-mechanical analysis of FGPM beams, most of them investigated geometrically linear response. Komijani et al. (2013a, 2013b) studied the nonlinear thermoelectrical stability and vibration of piezoelectric beams with graded properties under thermal and electrical loads. It is shown that due to the non-symmetric distribution of material properties in the thickness direction, the linear bifurcation buckling may not take place in this type of beams. Nonlinear stability and vibration analysis of temperature-dependent thermally pre-/post-buckled FGM beams over nonlinear hardening elastic foundation are reported by Esfahani et al. (2013). They used the generalized differential quadrature (GDQ) method to analyze the response of beams under two types of thermal loads, namely, uniform temperature rise and heat conduction across the thickness. Through various numerical studies, it is demonstrated that temperature-dependent material properties have a significant effect on the nonlinear thermal stability and vibration behavior of beams.

Beam elements are commonly used in micro- and nano-scale devices and systems, such as biosensors, atomic force microscopes, MEMS, and NEMS. In such applications, microstructure-dependent size effects are often significant. Since the conventional beam theories are not able to account for such size effects, many researchers have focused on developing beam models using continuum theories that contain additional material length scale parameters. Pure bending of a circular cylinder has been modeled by Anthoine (2000), employing the classical couple stress theory of Koiter (1964). Using the modified couple stress theory proposed by Yang et al. (2002), Ma et al. (2008, 2010, 2011) developed solutions for bending and vibration of Timoshenko and Reddy beam theories (Reddy, 1984, 2004a). A Timoshenko beam formulation for microstructure dependent functionally graded beams, based on a modified couple stress theory, has been recently presented by Reddy (2011) for bending, vibration, and buckling of beams. In this work the von Kármán simplification of the complete Green strain tensor is included to account for the geometric nonlinearity. Reddy and Berry (2012) presented a microstructure-dependent nonlinear theory for axisymmetric bending of circular plates, which accounts for through-thickness power-law variation of a two-constituent material and the von Kármán geometric nonlinearity. The classical and first-order shear deformation theories are considered in this study. Reddy and Arbind (2012) developed *algebraic relationships* for the deflections, slopes, stress resultants of the Timoshenko beam theory for microstructure-dependent FGM beams in terms of the same quantities of the conventional Euler–Bernoulli beam theory.

Most recently, Reddy and Kim (2012) and Kim and Reddy (2013) formulated a nonlinear modified couple stress-based third-order theory of functionally graded plates and obtained analytical solutions for bending, vibration, and buckling of FGM plates. Arbind et al. (2014) presented a finite element formulation for nonlinear analysis of microstructure dependent FGM beams based on the Euler–Bernoulli beam theory and the Timoshenko beam theory. The effect of nonlinearity, shear deformation, power-law index, microstructural length scale, and boundary conditions on the bending response of beams under mechanical loads is shown through various illustrations. Simsek and Reddy (2013a) presented bending and vibration studies of functionally graded microbeams using a new higher order beam theory and the modified couple stress theory, while Simsek and Reddy (2013b) developed a unified higher order beam theory for buckling of a functionally graded microbeam embedded in an elastic medium using a modified couple stress theory. Roque et al. (2013a) presented a study of a microstructure-dependent composite laminated Timoshenko beam using a modified couple stress theory and a meshless method. Roque et al. (2013b) analyzed Mindlin micro plates with a modified couple stress theory and a meshless method. Most recently, Gao et al. (2013) used Reddy's (1984) third-order theory to develop a non-classical third-order shear deformation plate model based on a modified couple stress theory, while Arbind et al. (2014) used the general third-order theory presented in Reddy and Kim (2012) to develop a modified couple stress-based third-order theory for nonlinear analysis of functionally graded beams and obtained analytical solutions for the linear case and finite element solutions for the nonlinear case. All these works used the modified couple stress theory proposed by Yang et al. (2002) and it contained a single length scale.