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Negative stiffness induced by shear along wavy interfaces

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ABSTRACT

The extension of an elastic body almost always leads to mechanical tension in the stretching direction. Here, we report an unusual phenomenon of global mechanical compression in the stretching direction of an elastic body containing sinusoidal wavy interfaces. When the elastic body with a wavy interface is subjected to tensile loading, the local stress state along the interface is mixed-mode. Finite element simulations show that the resistance of the interface to shear-slip locks the interface together, and generates a moment couple which rotates the interface. Once the local adhesive shear strength of the interface is reached, the interface slips and separates. Then, the rotated interface triggers a restoring moment couple which releases the stored elastic energy. The structure subsequently undergoes global compression in the stretching direction until the interface completely separates. This moment-couple-induced internal energy storage and release mechanism leads to a material structure that exhibits high initial strength and toughness, followed by post-peak compliant softening with negative stiffness. This structural negative stiffness behavior is closely-tied with the ability of the interface to store and release energy by rotation, and is also exhibited by polycrystalline structures where grain rotation is possible.

1. Introduction

The foundation of elasticity can be traced to the 17th century British physicist Robert Hooke who stated this Latin anagram "*ceiiinosssttuv*", whose solution he published as "*Ut tension, sic vis*" meaning, *"As the extension, so the force"*. This simple phase forms the basis for the definition of material stiffness, which almost always tends to be positive in order for a material to resist deformation. However, the concept of negative stiffness has been reported in pre-strained systems with stored elastic energy (Thompson, 1979). For example, negative axial stiffness was observed in lumped systems containing post-buckled viscoelastic rubber tubes (Lakes, 2001). Single cell tetrakaidecahedron models are known to exhibit a compressive force-deformation relation over a range of applied strains (Rosakis et al., 1993). The concept of negative stiffness is distinct from negative Poisson's ratio behavior, which refers to the transverse expansion of a material when stretched along the longitudinal axis (Lakes, 1987). In this work, we demonstrate that negative stiffness characteristics are exhibited by wavy interface structures bounded by elastic substrates.

Wavy interface architectures are commonplace in many biological and material systems. The extremely tough crustacean exoskeleton of lobster is made of wavy laminated structure called Bouligand layers (Raabe et al., 2005). In the mother-of-pearl nacre, which is the inner lining of shells of abalone, mussels and certain other mollusks, the wavy tablet microstructure has been shown to cause interlocking which leads to its amazing toughness (Barthelat et al., 2007).

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Experiments demonstrate that the incorporation of wavy interface structures in thin brittle films can substantially improve the interfacial toughness, presumably due to an extended path length for the growing crack, crack deflection, interface interlock, and enhanced plastic dissipation in the material (Sancaktar and Gomatam, 2001; Jennings, 1972; Mulville and Vaishnev, 1975). Such wavy interface architectures can be manufactured by etching, laser ablation, hot rolling, and grid blasting at the microscopic level, or by ion irradiation processes at the nanometer-scale (Kim et al., 2012). Numerical simulations suggest that increasing the amplitude of waviness of an interface can stabilize the propagation of cracks and prevent catastrophic failure (Zavattieri et al., 2007). In some cases, the interface does not unzip in a steady continuous manner but instead creates multiple damage zones ahead of the original crack, which further dissipates energy (Reedy, 2008).

Enhanced energy dissipation has also been reported for the bonding of a smooth soft substrate to a rough rigid surface. Studies have shown that the adhesion energy of the interface initially increases with roughness due to the increased contact area, provided that the soft surface can conformably deform to fit its rough counterpart (Briggs and Briscoe, 1977; Fuller and Roberts, 1981; Kim and Russell, 2001). Beyond a certain roughness limit, the work required for creating the surface undulations in the soft substrate cannot compensate for the adhesive bonding energy across the interfaces. Consequently, full contact between both surfaces can no longer be maintained, resulting in lowered adhesion (Fuller and Tabor, 1975). Recent studies reveal that surface roughness actually triggers micromechanical instabilities during the microscopic adhesion and decohesion processes, which can induce a several-fold increase in the dissipated energy compared to the intrinsic normal adhesion energy of the interface (Li and Kim, 2009; Guduru, 2007).

In the bonding of two deformable substrates, the local adhesion energy has both normal and shearing components. The energy of decohesion is therefore expected to come from the contributions of both the normal and shear adhesion energies of the interface. However, our finite element simulations show that the total dissipated energy is only the local normal adhesion energy. In addition, a regime of structural negative stiffness is exhibited by the wavy interface, resulting in global compression in the stretching direction. In clarifying these intriguing observations, we reveal that the shear-dominated loading along the wavy interface will induce a moment couple about the interface, which rotates the interface and builds up stored elastic energy in the structure. Under further tensile stretching, the interface slips and separates. Then, the rotated configuration of the interface induces global compression of the material system in the stretching direction, until all stored elastic energy is released by the restoring moment couple. This novel mechanism of internal energy storage and release results in significant initial strengthening and toughening of the interface, followed by post-peak negative stiffness response. Similar mechanism is observed during the deformation of polycrystalline structures where grain rotation is possible.

2. Problem formulation

Our model geometry consists of two semi-infinite linear elastic substrates, with elastic modulus E and Poisson's ratio ν, that are bonded along an interface with a sinusoidal morphology of amplitude A_0 and wavelength λ_0, as shown in Fig. 1a. A single period of the problem is modeled in the finite element method (FEM) using the general-purpose commercial finite element software Abaqus 6.11. Periodic boundary conditions are imposed by constraining the displacements of the individual nodes on the left end of the model to be the same as the displacements of the corresponding nodes on the right end of the model. The upper and lower boundaries of the model are subjected to uniform displacement loading in the x_2 direction, resulting in uniaxial strain conditions, until the interface completely separates. The material is homogeneous and isotropic for all simulations in this paper.

The finite element model is discretized into four-noded linear quadrilateral plane-strain elements, with four-noded cohesive elements placed along the sinusoidal interface joining the upper and lower substrates. The cohesive elements are implemented as user-defined elements which govern the relationship between the relative normal and tangential displacements Δ_n, Δ_t of the interface and the normal and tangential tractions T_n, T_t transmitted across the interface. These cohesive elements are governed by the reversible Xu and Needleman (1994) exponential traction–separation relationship depicted in Fig. 1b:

$$T_n(\Delta_n,\Delta_t)=\sigma_{max}\frac{\Delta_n}{\delta_n}\exp\left(1-\frac{\Delta_n}{\delta_n}\right)\left\{1-q\left[1-\exp\left(-\frac{\Delta_t^2}{\delta_t^2}\right)\right]\right\}$$

$$T_t(\Delta_n,\Delta_t)=2\sigma_{max}\left(\frac{\delta_n}{\delta_t}\right)\frac{\Delta_t}{\delta_t}q\left(1+\frac{\Delta_n}{\delta_n}\right)\exp\left(1-\frac{\Delta_n}{\delta_n}\right)\exp\left(-\frac{\Delta_t^2}{\delta_t^2}\right) \tag{1}$$

where $(\sigma_{max},\delta_n,\delta_t,q)$ are constitutive parameters. Under normal loading, the normal traction reaches a maximum value $T_n=\sigma_{max}$ at $\Delta_n=\delta_n$, and has a work of separation $\phi_n=\sigma_{max}\delta_n$ Eq. (1). Under simple shear, the tangential traction reaches a maximum value $T_t=\tau_{max}=\sqrt{2\exp(1)}q\sigma_{max}(\delta_n/\delta_t)$ at $\Delta_t=\delta_t/\sqrt{2}$, with the work of separation $\phi_t=q\phi_n$.

To quantify the *global* response of the wavy interface structure, we will homogenize the intrinsic traction-separation behavior along the wavy interface into an *equivalent* cohesive zone law along a planar interface as shown in Fig. 1a. Consider the linear elastic deformation fields $S[\sigma_{ij},u_j]$ for the real wavy interface system, and $\hat{S}[\hat{\sigma}_{ij},\hat{u}_j]$ for the equivalent planar