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Potential-based and non-potential-based cohesive zone formulations under mixed-mode separation and over-closure. Part I: Theoretical analysis

J. Patrick McGarry [a,*], Éamonn Ó Máirtín [a], Guillaume Parry [b], Glenn E. Beltz [c]

[a] *Mechanical and Biomedical Engineering, College of Engineering and Informatics, National University of Ireland, Galway, Galway, Ireland, United Kingdom*
[b] *SIMAP, Institut National Polytechnique de Grenoble, Grenoble, France*
[c] *Department of Mechanical Engineering, University of California Santa Barbara, Santa Barbara, USA*

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ABSTRACT

This paper presents a thorough analysis of potential-based and non-potential-based cohesive zone models (CZMs) under conditions of mixed-mode separation and mixed-mode over-closure. Problems are identified with the well established potential-based Xu–Needleman (XN) model and a number of new potential-based and non-potential-based models are proposed. It is demonstrated that derivation of traction–separation relationships from a potential function can result in non-physical repulsive normal tractions and instantaneous negative incremental energy dissipation under displacement controlled monotonic mixed-mode separation when the work of tangential separation exceeds the work of normal separation. A modified potential-based (MP) model is proposed so that the zone in which repulsive normal tractions occur can be controlled. The MP model also provides an additional benefit of correct penalisation of mixed-mode over-closure, in contrast to the XN model. In order to fully eliminate the problem of repulsive normal tractions a non-potential-based CZM (NP1) is also proposed. This model is shown to provide physically realistic behaviour under conditions of displacement controlled mixed-mode separation and over-closure. Noting that the form of the traction–separation equations differ for mode I and mode II separation for the XN, MP and NP1 models, an additional non-potential-based model (NP2) is proposed so that near mode-independent behaviour can be achieved in displacement controlled separation, while correctly penalising over-closure. Following from the NP2 model, a non-potential-based model in which coupling is based on the separation magnitude is considered (SMC model). In the final part of the paper the performance of each model under traction controlled mixed-mode separation is investigated by numerically inverting the traction–separation equations. Separation paths for the XN model reveal a strong bias toward mode I separation while the NP1 model exhibits a bias towards mode II separation. Interestingly, the NP2 model exhibits a high degree of mode sensitivity under traction controlled conditions, in contrast to its near mode independence under displacement controlled conditions. It is demonstrated that incorrect weighting of the coupling terms in non-potential models can lead to the existence of a singularity under traction controlled conditions. Finally, it is demonstrated that the potential-based models fail to capture a gradual change from mode II to mode I work of separation, as reported experimentally for traction controlled interface

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[*] Corresponding author. Tel.: +35 391 493 165.
E-mail address: patrick.mcgarry@nuigalway.ie (J.P. McGarry).

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separation. In a follow-on Part II companion paper a number of case studies are simulated, demonstrating that the theoretical findings of the present paper have significant implications for the finite element prediction of interface debonding.

1. Introduction

Cohesive zone models (CZMs) have been extensively used to describe the delamination process which occurs at the interface between two surfaces (Abdul-Baqi and Van der Giessen, 2001; Barenblatt, 1959; Camacho and Ortiz, 1996; Dugdale, 1960; Ural et al., 2009; Yan and Shang, 2009). CZMs have been used to model crack propagation in porous materials (Nakamura and Wang, 2001), ductile materials (Li and Chandra, 2003) and model coating failure in diamond-coated tools (Hu et al., 2008). CZMs can be coupled or uncoupled. In an uncoupled CZM, the normal traction is independent of the tangential opening separation and vice versa (Tijssens et al., 2000). Uncoupled CZMs are of limited use unless interface separation is constrained to occur in a single predefined direction (e.g. either mode I or mode II separation). Typically, however, CZMs are applied to engineering problems where the mode of interface separation is not predefined, requiring the use of mixed-mode formulations. Such mixed-mode applications require the use of a coupled CZM, whereby all components of the traction vector depend on both the normal and tangential interface separations, in order to provide a physically realistic response. For example if an interface undergoes a complete separation in the tangential direction, the resistance to a subsequent normal separation should be significantly reduced or eliminated. In the present study a detailed analysis of the performance of coupled CZMs under mixed-mode conditions is presented. Problems with existing models are identified and several new models are proposed to overcome such problems.

In addition to mixed-mode separation, mixed-mode over-closure must also be considered. The term over-closure refers to the phenomena whereby contacting surfaces penetrate into one another under a compressive contact stress. Clearly this is non-physical and should be correctly penalised by a CZM formulation. Most CZMs correctly penalise over-closure in pure mode I deformation, where no interface shear (tangential) displacement occurs. However, when the over-closure is mixed-mode, with both negative normal displacements and non-zero shear displacements, it is demonstrated that physically realistic over-closure behaviour is not trivially achieved by all CZM formulations.

The most commonly implemented coupled CZM is that developed by Xu and Needleman (XN) (1993) in which traction–separation relationships are obtained from the first derivatives of an interface potential function. In this model, normal and tangential behaviour is coupled via exponentially decaying functions of normal and tangential separation. The ratio of the work of tangential separation to the work of normal separation, commonly denoted using the symbol “q”, determines the relative strength of the interface under mode I and mode II separation. A similar potential-based model was also proposed by Beltz and Rice (1992), in which a sinusoidal tangential traction–separation relationship is coupled with a Xu–Needleman type normal traction–separation relationship. Again, the ratio of work of tangential separation to the work of normal separation provides a critical coupling parameter in this model. Several experimental studies have been reported in which the work of normal and tangential separation are different (Dollhofer et al., 2000; Warrior et al., 2003; Yang et al., 2001). However, the ratio of tangential to normal work, q, is arbitrarily set to unity for mixed-mode implementations of the XN CZM (Rahulkumar et al., 2000; Yuan and Chen, 2003; Zavattieri et al., 2008). It should also be noted that a number of previous studies adopt a value of $q \approx 0.43$ so that both the normal and tangential maximum tractions have the same value when the normal and tangential interface characteristic distances are assumed to be equal (Abdul-Baqi and Van der Giessen, 2002; Hattiangadi and Siegmund, 2005). A range of values of q ranging from 0.025 to 10.0 were considered in the original application of the XN model to debonding of spherical inclusions in a metal matrix composite (Xu and Needleman, 1993). In the study of thin film delamination, values of q ranging from 0.086 to 0.7 were considered by Abdul-Baqi and Van der Giessen (2001) while most recently a value of $q = 0.5$ was used by Yan and Shang (2009). A second coupling parameter used in the XN model is the traction-free normal separation following complete shear separation (commonly denoted using the symbol “r” in the XN potential function). It was noted by Abdul-Baqi and Van der Giessen (2001) that physically realistic penalisation of normal over-closure was computed only if $r \geq q$. However, the majority of studies that use the XN model set $r = 0$ (Rahulkumar et al., 2000; Xu and Needleman, 1993; Zavattieri et al., 2008). Further, a study by van den Bosch et al. (2006) suggests that physically realistic coupling is implemented by the XN model only when $q = 1$. An alternative non-potential-based cohesive zone formulation was proposed using the XN traction–separation equations when $q = 1$, with independent scaling factors then being applied to normal and tangential equations to account for differences in mode I and mode II interface strength.

To date, no comprehensive analysis of potential and non-potential-based CZMs has been published in which the complete range of diverse interface behaviour under mixed-mode separation and mixed-mode over-closure has been characterised. Furthermore, previous analyses of existing CZMs and proposed CZMs have relied on displacement controlled boundary conditions for model assessment (Abdul-Baqi and Van der Giessen, 2001; Mosler and Scheider, 2011; Park et al., 2009; van den Bosch et al., 2006), without considering traction controlled mode mixity. In Section 2.1 of the present paper an extensive analysis of the XN CZM under displacement controlled interface behaviour is presented, significantly expanding on the initial analyses of Abdul-Baqi and Van der Giessen (2001) and van den Bosch et al. (2006). It is