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Potential-based and non-potential-based cohesive zone formulations under mixed-mode separation and over-closure—Part II: Finite element applications

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A B S T R A C T

This paper, the second of two parts, presents three novel finite element case studies to demonstrate the importance of normal-tangential coupling in cohesive zone models (CZMs) for the prediction of mixed-mode interface debonding. Specifically, four new CZMs proposed in Part I of this study are implemented, namely the potential-based MP model and the non-potential-based NP1, NP2 and SMC models. For comparison, simulations are also performed for the well established potential-based Xu–Needleman (XN) model and the non-potential-based model of van den Bosch, Schreurs and Geers (BSG model). *Case study 1:* Debonding and rebonding of a biological cell from a cyclically deforming silicone substrate is simulated when the mode II work of separation is higher than the mode I work of separation at the cell-substrate interface. An active formulation for the contractility and remodelling of the cell cytoskeleton is implemented. It is demonstrated that when the XN potential function is used at the cell-substrate interface repulsive normal tractions are computed, preventing rebonding of significant regions of the cell to the substrate. In contrast, the proposed MP potential function at the cell-substrate interface results in negligible repulsive normal tractions, allowing for the prediction of experimentally observed patterns of cell cytoskeletal remodelling. *Case study 2:* Buckling of a coating from the compressive surface of a stent is simulated. It is demonstrated that during expansion of the stent the coating is initially compressed into the stent surface, while simultaneously undergoing tangential (shear) tractions at the coating-stent interface. It is demonstrated that when either the proposed NP1 or NP2 model is implemented at the stent-coating interface mixed-mode over-closure is correctly penalised. Further expansion of the stent results in the prediction of significant buckling of the coating from the stent surface, as observed experimentally. In contrast, the BSG model does not correctly penalise mixed-mode over-closure at the stent-coating interface, significantly altering the stress state in the coating and preventing the prediction of buckling. *Case study 3:* Application of a displacement to the base of a bi-layered composite arch results in a symmetric sinusoidal distribution of normal and tangential traction at the arch interface. The traction defined mode mixity at the interface ranges from pure mode II at the base of the arch to pure mode I at the top of the arch. It is demonstrated that predicted debonding patterns are highly sensitive to normal-tangential coupling terms in a CZM. The NP2, XN, and BSG models exhibit a strong bias towards mode I separation at the top of the arch, while the NP1 model exhibits a bias towards mode II debonding at the base of the arch. Only the SMC model provides mode-independent behaviour in the early stages of

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debonding. This case study provides a practical example of the importance of the behaviour of CZMs under conditions of traction controlled mode mixity, following from the theoretical analysis presented in Part I of this study.

1. Introduction

Cohesive zone models (CZMs) have been implemented for numerous applications. A large variety of cohesive zone laws have been reported in literature including; polynomial (Tvergaard, 1990), piece-wise linear (Tvergaard and Hutchinson, 1992), exponential (Xu and Needleman, 1993), and rigid-linear cohesive zone laws (Camacho and Ortiz, 1996). A CZM which first provided a phenomenological description of normal separation was developed by Needleman (1987) for analysing void nucleation by inclusion debonding in an elastic-viscoplastic matrix. A trapezoidal form of traction–separation response was used to model the resistance of crack growth following the initiation of a crack at an interface by Tvergaard and Hutchinson (1992). A piece-wise linear softening function was presented by Planas and Elices (1993) for the asymptomatic analysis of a cohesive crack. A rigid-linear law was implemented by Camacho and Ortiz (1996) to investigate the propagation of numerous cracks along random paths in a brittle material. Another study involving a rigid-linear CZM involved a damage-based CZM for simulating fatigue under cyclic loading (Ural et al., 2009).

The Xu–Needleman (XN) CZM was implemented by Abdul-Baqi and Van der Giessen (2001) for the prediction of indentation-induced delamination of an elastic thin film from a ductile substrate. It was reported that delamination initiated primarily in a tangential mode at two to three times the contact radius of the spherical indenter. Bilinear, exponential and trapezoidal CZMs were implemented by Yan and Shang (2009) to predict interfacial delamination in piezoelectric (PZT) thin films. Specifically, the fracture process along the Cr (chromium)/PZT interface was investigated. Modelling delamination of unidirectional fibre composites and adhesive joints, Sørensen et al. developed a mixed-mode cohesive zone from a potential surface using experimental data from double cantilever beam tests with uneven bending moments (Sørensen et al., 2008; Sørensen and Jacobsen, 2009).

While CZMs have been used extensively for a wide array of applications and loading conditions, few studies have rigorously investigated the effect of coupling terms on predicted behaviour, both for displacement and for traction controlled mode mixity. In Part I of this two parts study a thorough theoretical analysis of potential-based and non-potential-based CZMs under conditions of mixed-mode separation and mixed-mode over-closure was presented. Problems are identified with the well established potential-based Xu–Needleman (XN) model (Xu and Needleman, 1993) and a number of new potential-based and non-potential-based models are proposed. In particular, the following was demonstrated for the XN formulation: (i) Repulsive normal tractions are computed during mode II or mixed-mode separation if the work of tangential (mode II) separation (ϕ_t) exceeds the work of normal (mode I) separation (ϕ_n); (ii) Residual normal tractions must be overcome following a complete mode II or mixed-mode separation if ϕ_n exceeds ϕ_t; (iii) If $\phi_t = \phi_n$, incorrect penalisation of mixed-mode over-closure is computed, leading to reduced or repulsive tangential tractions; (iv) Under conditions of traction dependent mode mixity separation paths for the XN model reveal a strong bias toward mode I separation.

A modified potential-based (MP) formulation was proposed in order to partially overcome the limitations of the XN formulation. The MP model reduces the zone in which repulsive or residual tractions occur during mixed-mode separation if the work of normal and tangential separation are unequal. Additionally, the MP model improves upon the over-closure performance of the XN model by providing increased resistance to tangential separation during mixed-mode over-closure. While the MP formulation limits the zone in which non-physical repulsive and residual normal tractions occur, repulsive normal tractions cannot be fully eliminated in a potential-based CZM if $\phi_t \neq \phi_n$. An important consequence of this is that under traction controlled mode mixity when $\phi_t > \phi_n$, potential-based models fail to capture a gradual change from mode II to mode I work of separation, as reported experimentally for traction controlled interface separation (Hutchinson and Suo, 1992).

A non-potential-based formulation was proposed by van den Bosch et al. (2006) (BSG model) in order to provide improved coupling under mixed-mode separation. However, in Part I of this study it was noted that the BSG model does not provide correct penalisation of mixed-mode over-closure. A new non-potential-based formulation (NP1) was proposed in order to obtain correct coupling in both mixed-mode separation and in mixed-mode over-closure. Noting that it is not possible to prescribe identical mode I and mode II separation behaviour for the XN, BSG and NP1 formulations, a second non-potential-based formulation (NP2) was proposed. Identical traction–separation relationships in mode I, mode II and in pure mixed-mode (45°) separation are achieved for this formulation. This formulation also provides correct penalisation of mixed-mode over-closure. However, it was demonstrated that NP2 performs poorly in traction controlled mode mixity, with separation paths ultimately tending towards pure mode I or pure mode II with a singularity for 45° separation. Finally, following from the work of Tvergaard and Hutchinson (1993) a model in which the coupling terms are based on the separation magnitude (SMC model) was considered in order to provide mode-independent behaviour under displacement controlled conditions.