Optics Communications 316 (2014) 220–227

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Parabolic approximation analytical model of super-resolution spot generation using nonlinear thin films: Theory and simulation

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ARTICLE INFO

Article history:
Received 28 October 2013
Received in revised form
25 November 2013
Accepted 2 December 2013
Available online 17 December 2013

Keywords:
Optical nonlinearity
Diffraction limit
Super-resolution

ABSTRACT

In this work, we present a theoretical analytical model called the parabolic approximation analytical model that considers the absorption coefficient and refractive index in a parabolic approximation profile along the radial direction. By using this model the generation of a super-resolution spot is analyzed in detail, and a numerical simulation subsequently conducted to verify the analytical model. This work is helpful in understanding the super-resolution effect of nonlinear thin films.

1. Introduction

Overcoming the Abbe diffraction limit and obtaining a super-resolution spot have been hot topics because of the important demands in the fields of nanolithography, ultrahigh density data storage, and nanoscale-resolved optical imaging and detection. Numerous methods and techniques have been proposed to overcome the Abbe limit, including the use of a near-field probe [1], surface plasmon effect [2–5], solid immersion lens [6], phase modulation filter [7,8], metamaterial lens [9], microsphere based microscopic-lens [10], and fluorescence labeling [11], among others.

One promising approach involves taking advantage of nonlinear absorption and refraction to break the diffraction limit and form a super-resolution spot are good physical ideas. In recent years, numerous physical models on nonlinear super-resolution have been proposed by our research group and other researchers, including the self-focusing effect-induced super-resolution [12–15], internal multi-interference reshaping-induced super-resolution [16], nonlinear saturation absorption-induced aperture-type super-resolution [17–19], and nonlinear reverse saturation absorption or multi-photon absorption-induced super-resolution energy absorption spot and lithography [20,21]. Experimental applications in optical data storage and nanolithography have been reported [22–27]. However, the detailed physical process of super-resolution spot generation remains unresolved.

When a Gaussian laser beam spot focused onto thin films with nonlinear absorption and refraction, some physical effects need to be considered. First, the absorption coefficient α and refractive index n are functions of the laser intensity. Second, a positive nonlinear refraction can cause a self-focusing spot, which is smaller than the incident spot, whereas a negative nonlinear refraction causes a self-defocusing spot, which is larger than the incident spot. Third, a negative nonlinear absorption can induce the formation of a below-diffraction limited aperture and generate a super-resolution spot, whereas a positive nonlinear absorption has an opposite effect. In the present study, a comprehensive analytical model called the parabolic approximation analytical model, is constructed by considering the absorption coefficient and refractive index in a parabolic approximation profile along the radial direction. The generation of super-resolution spot is analyzed in detail, and a numerical simulation is subsequently conducted to verify the analytical model. This work is helpful in understanding the super-resolution effect of nonlinear thin films.

2. Parabolic approximation analytical model

A collimated Gaussian beam is focused onto nonlinear thin films (Fig. 1). The intensity profile of the focused spot can be written as

$$I_{inc}(r) = I_0 \exp\left(-\frac{2r^2}{w_0^2}\right) \tag{1}$$

where w_0, r, and I_0 are the spot radius, radial coordinate, and peak intensity at $r=0$, respectively. The focused spot is incident to the

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http://dx.doi.org/10.1016/j.optcom.2013.12.013

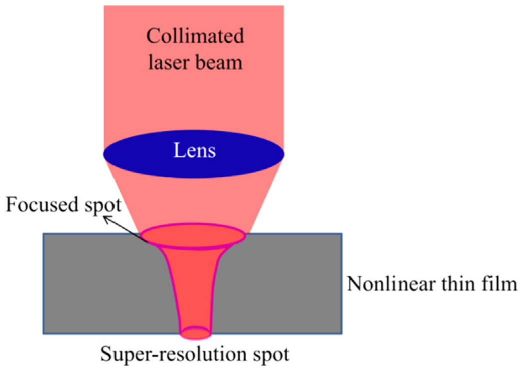

Fig. 1. Schematic of super-resolution generation using nonlinear thin films.

nonlinear thin films and causes nonlinear absorption and nonlinear refraction, and the nonlinear index to be a function of the laser intensity.

Assuming that α and n remain unchanged at the direction of the thin film thickness z, n and α can be expressed using Eq. (6) as follows

$$\alpha = \alpha_0 + a_I I = \alpha_0 + a_I I_0 \exp\left(-\frac{2r^2}{w_0^2}\right) \quad (2)$$

$$n = n_0 + n_I I = n_0 + n_I I_0 \exp\left(-\frac{2r^2}{w_0^2}\right) \quad (3)$$

where α_0 and α_I are the linear and nonlinear absorption coefficients, respectively and n_0 and n_I are the linear refractive index and nonlinear refraction coefficient, respectively.

To simplify the analysis, the exponential terms of Eqs. (2) and (3) are calculated using the Taylor expansion to yield

$$\exp\left(-\frac{2r^2}{w_0^2}\right) = b_0 + b_1\left[\frac{r}{w_0}\right]^2 + \cdots \quad (4)$$

where $b_0 = 1$, and $b_1 = -2$. Thus, Eqs. (2) and (3) can be rewritten as

$$\alpha \approx \alpha_c(1+\beta r^2) \text{ at } |\beta| r^2 \ll 1 \quad (5)$$

with

$$\alpha_c = \alpha_0 + a_I I_0 \quad (6)$$

$$\beta = -\frac{2a_I I_0}{\alpha_c w_0^2} \quad (7)$$

$$n \approx n_c(1+\gamma r^2) \text{ at } |\gamma| r^2 \ll 1 \quad (8)$$

with

$$n_c = n_0 + n_I I_0 \quad (9)$$

$$\gamma = -\frac{2n_I I_0}{n_c w_0^2} \quad (10)$$

Eqs. (5) and (8) indicate that for nonlinear thin films, n and α profiles along the radial coordinate r can be expressed in a parabolic approximation form. The parabolic approximation treatment is suitable for lens-like and (reverse) saturated absorption media. The complex refractive index can be written as

$$\tilde{n} = n - i(\alpha/2k) \quad (11)$$

where $k = 2\pi/\lambda$ is the wavenumber. The focused spot travels in the nonlinear thin films, and the electric field E follows the Helmholtz function

$$\Delta E + k^2\tilde{n}^2 E = 0 \quad (12)$$

Based on Eqs. (5), (8), and (11), $\tilde{n}^2$ is expressed as

$$\tilde{n}^2 = \left[n_c(1+\gamma r^2) - i(a_c/2k)(1+\beta r^2)\right]^2 \quad (13)$$

By mathematical operations and ignoring the terms $(\beta r^2)^2$ and $(\gamma r^2)^2$ because $|\gamma| r^2 \ll 1$ and $|\beta| r^2 \ll 1$, Eq. (13) is further simplified as

$$\tilde{n}^2 \approx (n_c - i(a_c/2k))^2\{1 - [-2\gamma + i(2a_c(\beta-\gamma)/(2kn_c - ia_c))]r^2\} \quad (14)$$

Substituting Eq. (14) into Eq. (12) yields

$$\Delta E + k^2\tilde{n}_c^2(1-\Gamma^2 r^2)E = 0 \quad (15)$$

with

$$\tilde{n}_c = n_c - i(a_c/2k) \quad (16)$$

$$\Gamma^2 = -2\gamma + i(2a_c(\beta-\gamma)/(2kn_c - ia_c)) \quad (17)$$

where Γ is the propagation constant. Let E be in the form

$$E(r,z) = A(r,z)\exp(-ik\tilde{n}_c z) \quad (18)$$

By substituting Eq. (18) into Eq. (15), in cylindrical coordinates (r,φ,z) one can get

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{1}{r^2}\frac{\partial^2 A}{\partial \varphi^2} + \frac{\partial^2 A}{\partial z^2} - 2ik\tilde{n}_c\frac{\partial A}{\partial z} - k^2\tilde{n}_c^2\Gamma^2 r^2 A = 0 \quad (19)$$

For rotational symmetries condition, the amplitude distribution $A(r,z)$ is not dependent on φ, that is $\partial^2 A/\partial\varphi^2 = 0$, thus Eq. (19) is rewritten as

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} - 2ik\tilde{n}_c\frac{\partial A}{\partial z} - k^2\tilde{n}_c^2\Gamma^2 r^2 A = 0 \quad (20)$$

In the range of light wavelength of optical axis if the following formulas meet

$$\left|\frac{\partial A(z)}{\partial z}\right| \ll |kA(z)|, \qquad \left|\frac{\partial^2 A(z)}{\partial z^2}\right| \ll \left|k\frac{\partial A(z)}{\partial z}\right| \quad (21)$$

Then a slowly varying approximation can be used, Eq. (20) is rewritten as [28]

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} - 2ik\tilde{n}_c\frac{\partial A}{\partial z} - k^2\tilde{n}_c^2\Gamma^2 r^2 A = 0 \quad (22)$$

The trial solution of Eq. (22) is set as follows

$$A(r,z) = \frac{A_0}{s(z)}\exp\left[\frac{-ik\tilde{n}_c r^2}{2q(z)}\right] \quad (23)$$

By substituting Eq. (23) into Eq. (22) and in accordance with the condition of an arbitrary r, the differential equations are obtained as follows

$$\frac{dq}{dz} = 1 + \Gamma^2 q^2 \quad (24)$$

$$\frac{1}{s}\frac{ds}{dz} = \frac{1}{q} \quad (25)$$

From Eq. (17), Γ is independent of the axial coordinate z, and the boundary condition is

$$q(z)|_{z=0} = s(z)|_{z=0} = q_1 \quad (26)$$

where $z = 0$ is the beam waist position of the incident Gaussian light beam. Thus the solutions of Eqs. (24) and (25) are derived as follows

$$q(z) = \frac{q_1 \cos\Gamma z + \frac{1}{\Gamma}\sin\Gamma z}{-q_1\Gamma\sin\Gamma z + \cos\Gamma z} \quad (27)$$

$$s(z) = \frac{1}{\Gamma}\sin\Gamma z + q_1\cos\Gamma z \quad (28)$$