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# A variational approach to the fracture of brittle thin films subject to out-of-plane loading

A. Mesgarnejad [a], B. Bourdin [b], M.M. Khonsari [a,*]

[a] Department of Mechanical Engineering, Louisiana State University, Baton Rouge, LA 70803, USA
[b] Department of Mathematics and Center for Computation & Technology, Louisiana State University, Baton Rouge, LA 70803, USA

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ABSTRACT

We address the problem of fracture in homogenous linear elastic thin films using a variational model. We restrict our attention to quasi-static problems assuming that kinetic effects are minimal. We focus on out-of-plane displacement of the film and investigate the effect of bending on fracture. Our analysis is based on a two-dimensional model where the thickness of the film does not need to be resolved. We derive this model through a formal asymptotic analysis. We present numerical simulations in a highly idealized setting for the purpose of verification, as well as more realistic micro-indentation experiments.



## 1. Introduction

Thermal barrier coatings, thin-lubricant films, and electronic display devices are examples of applications in which the integrity of mechanical components depends largely on the integrity of a thin film of material applied on the surface of a substrate. The need to gain insight into the nature of thin films under various thermal and mechanical loadings has led to a large body of theoretical, experimental and numerical publications reviewed in detail in Mishnaevsky and Gross (2004) and Lawn et al. (2002). Classical fracture mechanics has been widely used in the past few decades, and the majority of published works deal with the determination of critical loading for a pre-existing crack, usually growing on a pre-defined path. Such an assumption may be too restrictive when dealing with real life applications in which the nucleation point may be unknown and multiple cracks may be interacting or growing along unknown paths. Fig. 1, for instance, shows fracture patterns obtained during micro-indentation experiments and illustrates how qualitatively and quantitatively different crack patterns arise for different scales or material properties.

To treat the problem of pre-tensioned films subject to in-plane displacements, Hutchinson and Suo (1992) introduced a non-dimensional fracture driving force $Z = G/E_e$, where $G$ is the elastic energy release rate and $E_e$ is the stored elastic energy per unit volume of the material. Using this parameter, they were able to categorize different fracture patterns in thin films and, more specifically, showed cases where surface cracks occur or a network of channel fractures develops. More recently, in Xia and Hutchinson (2000) developed a two-dimensional membrane model and derived solutions for a single crack and a network of parallel cracks, as well as spiral cracks based on linear fracture mechanics. Some of these solutions were recovered through a variational approach in Léon Baldelli et al. (2013).

* Corresponding author. Tel.: +1 2255789192; fax: +1 2255785924.
*E-mail addresses:* amesga1@tigers.lsu.edu (A. Mesgarnejad), bourdin@lsu.edu (B. Bourdin), khonsari@me.lsu.edu (M.M. Khonsari).

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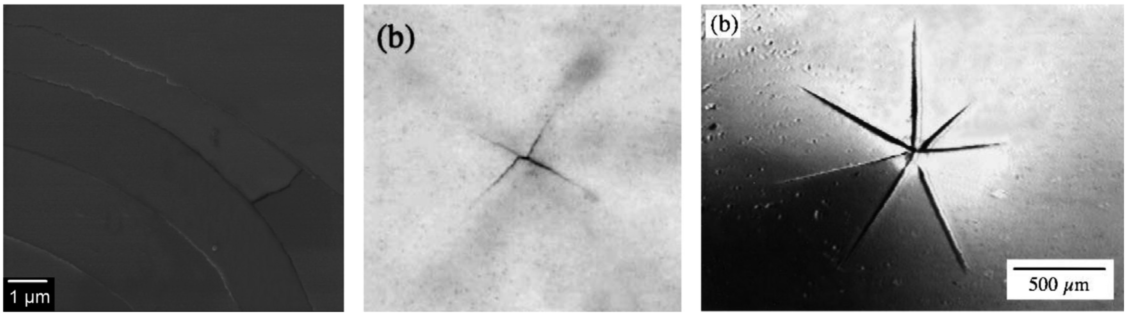


**Fig. 1.** Micro-indentation experiments reproduced from Sierros et al. (2011, Figure 3) (left) and Lawn et al. (2002, Figure 7–8) (center, right); reproduced with permission.

Our aim in this article is to examine the fracture of thin films with negligible thickness compared to the dimensions of the domain being analyzed, and in which transverse cracks span the entire cross-section of the film. We introduce a bulk energy consisting of two terms: the energy stored in the thin film and a Winkler foundation-type (Szilard, 2004) energy due to deformation in the bond between the thin film and the substrate. We justify these under specific scaling properties of the thickness and elastic properties of the film and bonding layers by an asymptotic analysis argument. This reduces the settings of our problem from three to two dimensions.

We propose to adopt the point of view of the variational approach to fracture mechanics (Francfort and Marigo, 1998; Bourdin and Chambolle, 2000; Bourdin, 2007; Bourdin et al., 2008), which we adapt to our specific situation, in order to eliminate the reliance on *a priori* knowledge of the crack path or morphology. We build upon the work of Léon Baldelli et al. (2013), but focus on the out-of-plane deformation of a film perfectly bonded to an elastic substrate. The postulated evolution law is based on sequences of *unilateral global minimization* of a total energy consisting of the sum of a bulk energy associated with the elastic deformation of the thin film away from cracks and the surface energy due to creation of transverse cracks. The assumption of cracks propagating in a quasi-static setting is consistent with our focus on the asymptotic limit of a film of vanishing thickness and on cracks that are long compared to the thickness of the film.

We propose a numerical approach based on a regularized energy similar to the one presented in Bourdin and Chambolle (2000). To verify our approach we focus on highly idealized situations, in particular in one-dimensional cases where exact solutions can be built. Using this relatively simple model, we are able to highlight several observed behaviors of cracks in thin films, including the nucleation of arrays of parallel cracks (see Sections 3 and 4.1), fracture branching, cell formation, and formation of networks of channel cracks (see Section 4.2).

The article is organized as follows. In Section 2.1, we give the elastic and fracture energies for a static problem, and derive a model for quasi-static evolutions in Section 2.2. In Section 2.3, we propose a non-dimensional formulation. In Section 2.4, we present our numerical approach. Section 3 is devoted to the verification of the numerical implementation in an idealized setting. In Section 4, we offer two more realistic numerical experiments highlighting the versatility of our formulation. Additionally, a numerical approach leading to an exact solution of the one dimensional problem is presented in Appendices A and B is devoted to the formal derivation of our reduced model.

## 2. Variational model for fracture of a thin film

### 2.1. Formulation of the problem

A host of problems arises in applications that are based on a reduced dimensional formulation. Plate and shell models, in theory of elasticity, are examples of such a dimension reduction. Here we are interested in one such problem with an elastic homogenous thin layer bonded to a substrate. For the engineering minded reader, this model is similar to a plate with an elastic foundation (Szilard, 2004).

We consider an elastic thin film bonded to the upper surface $\Omega \subset \mathbb{R}^2$ of the substrate $\mathcal{W} \subset \mathbb{R}^3$ by a Winkler type foundation. We denote the thin film's domain $\Omega_f = \Omega \times (0,h) \subset \mathbb{R}^3$. We focus on channel cracks $\Gamma_f = \Gamma \times (0,h) \subset \mathbb{R}^2$ in the thin film. We consider loading through an imposed displacement at upper surface of the substrate namely $w_t = w|_{\Omega \times \{0\}}$. Intuitively, it is reasonable to assume that the thin film does not carry any vertical load and that its deformation is driven by the movement of substrate (Hutchinson and Suo, 1992). In all that follows, the displacement of the substrate–film interface is supposed to be known *a priori*.

To use the variational approach to fracture mechanics, the potential energy of the system must be calculated. Our model applies to situations where the dominant term in the elastic energy comes from bending effects. We account for a simplified configuration where the cohesive bond between the film and the substrate acts as an elastic highly anisotropic (essentially one-dimensional) medium (*i.e.,* a Winkler foundation). Rigorous validation of such a formulation requires examination of the three-dimensional elastic energies of the film and that of the cohesive bond when thicknesses of both layers approach zero. In fact, it is possible to rigorously derive the two-dimensional problem as a limit of a three-dimensional energy when