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An integral split-step fourier method for digital back propagation to compensate fiber nonlinearity

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ABSTRACT

In optical fiber transmission systems, the split-step Fourier method (SSFM) has been widely used in digital back propagation (DBP) to compensate fiber nonlinearity. In this paper, by using the Lagrange's Integral Mean Value Theorem (LIMVT), we derive an analytical expression to calculate the optimal value of the nonlinearity calculation position (NLCP) for different systems and we propose an integral SSFM (I-SSFM) based on the expression. The I-SSFM can be performed more accurately and efficiently without parameter optimization. Simulations of various transmission links show that the I-SSFM outperforms the conventional asymmetric SSFM (A-SSFM) and the symmetric SSFM (S-SSFM) significantly, especially when we employ less amount of steps to ensure computation efficiency. The computation effort of the I-SSFM reaches as low as 50% of that of the S-SSFM.

1. Introduction

In recent years, real-time 40 Gb/s [1] and 100 Gb/s [2,3] coherent optical transmission systems have been demonstrated. To achieve even higher bit-rate and spectral efficiency, multilevel modulation formats such as 16-level quadrature amplitude modulation (16QAM) and 64QAM are to be employed. However, systems employing higher-level modulation formats require higher SNR and hence higher launch powers and this can significantly reduce the possible transmission distance and capacity due to fiber nonlinear impairments [4,5]. Therefore, mitigation or compensation of fiber nonlinearity becomes significant and has been widely studied, among which the digital signal processing (DSP) is a promising solution.

Of all the approaches employing DSP techniques to suppress the fiber nonlinearity, the digital back propagation (DBP) [6–9] shows the best performance and has become the benchmark for fiber nonlinearity compensation. By employing the split-step Fourier method (SSFM), DBP can solve the nonlinear Schrödinger equation (NLSE) inversely and hence the optical signals can be reconstructed at the receiver side.

Generally, there are 2 ways to realize the SSFM, i.e., the asymmetric SSFM (A-SSFM) and the symmetric SSFM (S-SSFM). The A-SSFM performs one linear and then one nonlinear operation sequentially in each step and is not iterative. But to enhance the calculation accuracy, the S-SSFM calculates half a linear and one nonlinear followed by the rest half a linear operation sequentially in each step and often utilizes two additional iterations [6,11]. The non-iterative A-SSFM saves computation efforts, but the iterative S-SSFM shows a better performance at a cost of higher implementation complexity.

To improve the computation efficiency without significant performance degradation, a modified non-iterative SSFM (M-SSFM) for DBP was proposed in [10]. This proposal improves the performance of DBP by shifting the nonlinearity calculation position (NLCP) without additional iterations [10]. The M-SSFM outperforms the A-SSFM and is less complex than the S-SSFM. However, to the best of our knowledge, no analytical expression for the NLCP has been reported, which means that it is difficult to determine the optimal value of NLCP in actual systems with various parameters, like the dispersion and launch power. In [10] the optimum of NLCP is obtained by enumerating and testing different values of NLCP. Still, the M-SSFM is often combined with parameter optimization [10], bringing extra work in obtaining the optimized parameters for different systems.

In this paper, we reveal why better performance can be achieved by shifting the NLCP and we derive an analytical expression to calculate the optimal value of NLCP. This expression is derived with the Lagrange's Integral Mean Value Theorem (LIMVT) and is applicable to systems with different parameters, like the dispersion, transmission length and launch power. And based on the expression, we propose an integral SSFM (I-SSFM). Unlike the M-SSFM, the resulting expression allows us to obtain the optimum of NLCP

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accurately and the I-SSFM can be performed without optimizing the parameters, avoiding the extra work brought by parameter optimization. Through simulations of transmission links of EDFA systems with various parameters, we investigate the performance of the I-SSFM employing the derived expression and the applicability of our proposal for different systems.

2. Theory

2.1. Back propagation theory

Ignoring the higher order linear and nonlinear terms, the NLSE for a single channel optical system is [11]

$$\frac{\partial A(z,\ T)}{\partial z}=(\widehat{D}+\widehat{N})A(z,\ T) \tag{1}$$

where A is the optical field envelope. $\widehat{D}$ and $\widehat{N}$ are the linear and nonlinear operators respectively and defined as

$$\widehat{D}=-\frac{\alpha}{2}-\frac{i\beta_2}{2}\frac{\partial^2}{\partial T^2} \tag{2}$$

$$\widehat{N}=i\gamma|A(z,\ T)|^2 \tag{3}$$

where α is the fiber attenuation, β_2 is the group velocity and γ is the fiber nonlinear coefficient.

The DBP attempts to back propagate the received signals in a virtual fiber by taking the inverse spatial evolution of Eq. (1)

$$\frac{\partial A(z,\ T)}{\partial z}=(\widehat{D}^{-1}+\widehat{N}^{-1})A(z,\ T) \tag{4}$$

here $\widehat{D}^{-1}=-\widehat{D}$ and $\widehat{N}^{-1}=-\widehat{N}$ are the inverse operators. The parameters used in Eq. (4) are opposite to the fiber parameters in sign, i.e., $(-\alpha, -\beta_2, -\gamma)$. Fig. 1 shows the process of the forward propagation (FP) and the digital back propagation (DBP).

We often numerically solve the NLSE by employing the most commonly used S-SSFM

$$A(z+h,T)\approx \exp\left[\frac{h}{2}\widehat{D}\right]\exp\left[\int_z^{z+h}\widehat{N}(s)\,ds\right]\exp\left[\frac{h}{2}\widehat{D}\right]A(z,\ T) \tag{5}$$

$$\int_z^{z+h}\widehat{N}(s)\,ds\approx\frac{h}{2}[\widehat{N}(z)+\widehat{N}(z+h)] \tag{6}$$

where h is the step size. The integral for $\widehat{N}(s)$ is generally approximated to Eq. (6) by the trapezoidal rule and two iterations are often used [6,11]. The accuracy of the iterative S-SSFM improves with the increase of the number of iterations or the increase of the number of steps per span, both increasing the computation efforts.

2.2. The calculation of NLCP

Our algorithm is based on the assumption that the optical signal power evolves exponentially through the propagation in the fiber. So first it is necessary to investigate how the signal power evolves through transmission in systems both with and without dispersion in the FP.

First let us consider the fiber without dispersion. Provided that no dispersion exists in the fiber, then there is only nonlinear effect. As the fiber nonlinearity essentially only changes the phase of every symbol, it does not affect the signal's power. So the optical signal power theoretically follows the exponential decaying under this scenario.

But for systems with dispersion, there is inter-symbol-interference (ISI) and the optical signal power may not simply decay exponentially. To investigate how the signal power evolves when both dispersion and nonlinear effect exist, we first divide the constellation into 3 circles (C1, C2 and C3) according to the power of the modulated symbols as shown in Fig. 2. Before transmission, symbols belonging to the same circle have the same power. Then we simulate a single-channel single-polarization 16QAM transmission system with a symbol rate of 28 Gbaud (112 Gbps). We consider the standard single mode fiber (SSMF) with attenuation $\alpha=0.2$ dB/km and nonlinearity coefficient $\gamma=1.32\ \mathrm{W}^{-1}\ \mathrm{km}^{-1}$. The input power is fixed at -3 dBm and transmission length is fixed at 8×80 km. The noise figure (NF) of the EDFA is 4 db and 16,384 symbols are transmitted. Two dispersion coefficients of $D=4.4$ ps/nm/km and $D=16$ ps/nm/km are employed. We record the power of each symbol at every step in every loop. Then we calculate the average power of the symbols belonging to the same circle at every step in every loop. The result is shown in Fig. 3(a) and (b). We can see that from the second loop on, the average power of the 3 circles all approximate exponential decaying for both systems with $D=4.4$ and $D=16$.

Though we conduct simulations with only several typical parameters, to some extent it validates our assumption of exponential decaying in the FP is acceptable from the viewpoint of simulation. As the DBP is essentially a reverse calculation of the FP, the signal power in DBP should also evolve exponentially. Then optical signal power in each span in the FP and DBP should evolve approximately as the exponential curves shown in Fig. 4.

Now we assume that the S-SSFM is performed once per span when using the DBP, so we will have that the step size $h=l_{span}$. Then we calculate half a linear, one nonlinear and then half a linear operation sequentially. For the nonlinear calculation, we

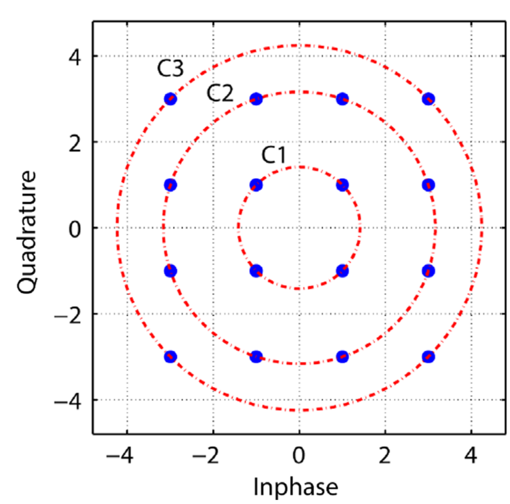

Fig. 2. The division of 3 circles for the constellation.

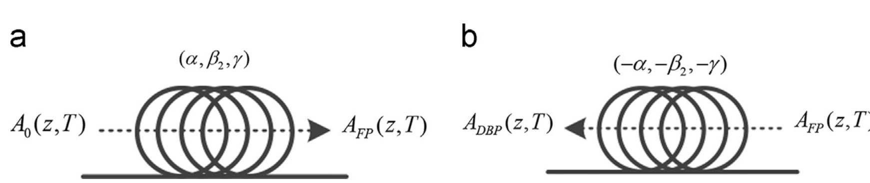

Fig. 1. (a) Forward propagation (FP) process through the fiber. (b) Digital back propagation (DBP) propagation through the virtual fiber.