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Microscopic theory of photonic band gaps in optical lattices

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ABSTRACT

We propose a microscopic model to describe the scattering of light by atoms in optical lattices. The model is shown to efficiently capture Bragg scattering, spontaneous emission and photonic band gaps. A connection to the transfer matrix formalism is established in the limit of a one-dimensional optical lattice, and we find the two theories to yield results in good agreement. The advantage of the microscopic model is, however, that it suits better for studies of finite-size and disorder effects.

1. Introduction

When an electromagnetic wave is sent into an atomic cloud, the interference of the radiation fields emitted by every atom gives rise to cooperative scattering. In disordered systems this interference phenomenon was first described by Dicke who evidenced the superradiant emission of the cloud [1]. Later, other striking features linked to cooperative scattering have been observed, such as the collective Lamb shift [2–4] and a reduction of the radiation pressure force [5,6]. The search for the localization of light by the disorder itself is still ongoing [7,8].

The hallmark of the photonic properties of *ordered atomic ensembles*, such as optical lattices, is the formation of a band structure similar to those encountered in photonic crystals. Photonic band gaps (PBGs) have been predicted in one-dimensional arrays of atomic clouds using the transfer matrix (TM) technique [9]. This approach, which relies on the description of an atomic ensemble as a continuous dielectric with a very large transverse size, describes well the situation of recent experiments [10–12], which culminated in the first observation of a PBG in a one-dimensional optical lattice [13,14].

In the case of three-dimensional optical lattices, the Bloch–Floquet model has been used to calculate the propagation of electromagnetic modes in Fourier space and identify omnidirectional PBGs in certain geometries assuming infinite and perfectly periodic lattices [15,16]. Nevertheless, omnidirectional PBGs remain to be observed experimentally.

2. Microscopic model

In this paper we propose a microscopic model of cooperative scattering from an ordered atomic gas, treating the atoms as point-like scatterers interacting with light via an internal resonance. We show that this model is able to describe the opening of a forbidden photonic band due to multiple reflection of light between adjacent lattice sites. We support our assertion in two ways. Using numerical simulations of the microscopic model we find that Bragg scattering and PBGs arise in our system. We also demonstrate that under a coarse-graining hypothesis and in the limit of a one-dimensional optical lattice, the microscopic model boils down to the TM formalism used, e.g., in Refs. [9,12,13].

It must be highlighted that our microscopic model does not contain the limitations of the above-mentioned other techniques. In particular, it does not reduce the atomic layers to a smooth dielectric, nor does it assume the atomic cloud to be perfectly periodic or infinite in any direction. It is thus notably suited to study the role of the disorder and finite-size effects on photonic bands.

The collective light scattering by an atomic ensemble is described by the following coupled equations [17–19]:

$$\left(i\Delta_0-\frac{\Gamma}{2}\right)\beta_j=\frac{i\wp}{2\hbar}E_0(\mathbf{r}_j)+\frac{\Gamma}{2}\sum_{k\neq j}\frac{\exp(ik_0|\mathbf{r}_j-\mathbf{r}_k|)}{ik_0|\mathbf{r}_j-\mathbf{r}_k|}\beta_k \tag{1}$$

where $\mathbf{r}_j$ is the position of the jth atom and β_j is the excitation amplitude of its dipole. The first term on the right-hand side of Eq. (1) corresponds to the field $E_0(\mathbf{r})$ of the incident laser beam, whereas the last term characterizes the radiation from all other atoms. Δ_0 is the detuning of the incident laser with respect to the

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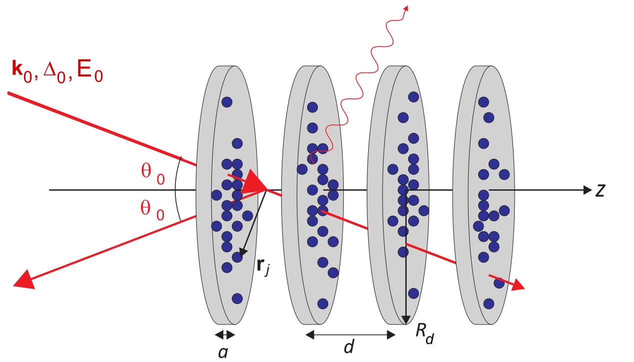

Fig. 1. Experimental setup: an array of disks randomly filled with atoms is irradiated from the side by a probe beam incident under an angle θ_0. The system can be considered as one-dimensional assuming that $a, d \ll R_d$, where a and R_d are the thickness and the radius of each disk, respectively, whereas d is the inter-layer distance.

atomic transition, Γ is the single atom spontaneous decay rate and $\wp$ is the electric dipole matrix element.

We test our model on a one-dimensional periodic stack of N_d parallel disks randomly filled with N_a atoms each, illuminated by a laser beam incident under an angle θ_0 with respect to the lattice axis (see Fig. 1). We compare the predictions of the microscopic model with those of the TM formalism noting that, while the TM approach assumes a radially infinite extension of the disks, our model is able to account for any distribution, e.g., the Gaussian distribution common for thermal atomic clouds.

We emphasize that Eq. (1) describes the scalar light scattering. We also present a full vectorial model in Section 5, giving the results in very good agreement with the scalar one derived for the one-dimensional lattice geometry.

3. Bragg scattering

We first investigate the scattering properties of our system under the Bragg condition, which means that the phase-shift of the incident wave between two successive atomic disks is π. In this case, the interference of the waves reflected from each disk is constructive, and the system is a Bragg reflector. This property is well reproduced by our model in which, despite the point-like nature of the scatterers, the incident Gaussian beam is reflected by the atomic structure (see Fig. 2), where we consider ^{85}Rb atoms interacting with the light fields via their $D2$ line. The total electric field E is given by the sum of the incident field $E_0(\mathbf{r})$ and the scattered field

$$E_{scat}(\mathbf{r}) = -\frac{\hbar\Gamma}{\wp}\sum_j \beta_j \frac{\exp(ik_0|\mathbf{r}-\mathbf{r}_j|)}{k_0|\mathbf{r}-\mathbf{r}_j|}, \tag{2}$$

and according to the extinction theorem, the lattice produces in the forward direction a field opposed to the incident one. This demonstrates the suitability of our model to study, e.g., the microscopic version of the Ewald–Oseen theorem [20].

It can be observed that not all incident light is reflected by the atomic structure. A significant part of it is re-emitted in the form of the spontaneous emission. This phenomenon, which is normally captured in the imaginary part of the refractive index, is naturally present in the microscopic model (1). The spontaneous emission appears in Fig. 3 as the radiation into non-paraxial modes. It should be noted that our microscopic model does not contain light absorption, and we have verified that it conserves energy, i.e., pursuant to Maxwell's equations, the light which is not reflected or transmitted is spontaneously scattered more or less isotropically. The deviation from perfect isotropy, visible in Fig. 3 as angular fluctuations, is a signature of the disorder existing in each atomic disk.

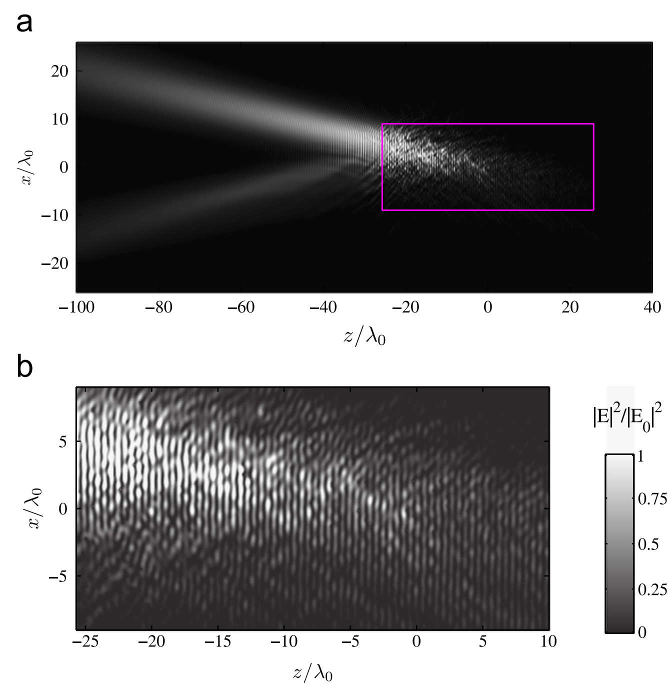

Fig. 2. The above picture: Intensity of the light in the $y = 0$ plane as it enters a one-dimensional optical lattice and its reflection. The rectangle marks the limit of the atomic structure. Below: Zoom of the left part of the atomic lattice. The luminous grains correspond to the strong field radiated by the atoms close to the $y = 0$ plane. The simulations are realized for $N = 9000$ atoms randomly distributed over $N_d = 100$ layers of thickness $a = 0.06\lambda_0$ and radius $R_d = 9\lambda_0$, the distance between the atomic disks is $d = 0.508\lambda_0$ with λ_0 being the resonance wavelength. The Gaussian beam of waist $4.5\lambda_0$ and power 100 mW is detuned by $\Delta_0 = \Gamma$ from the atomic transition and creates the angle $\theta_0 = 0.2$ rad with the lattice axis.

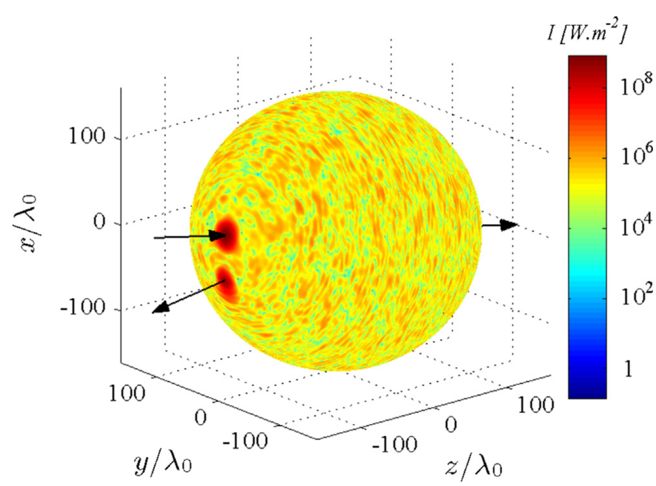

Fig. 3. Far-field intensity I at the distance $150\lambda_0$ from the lattice. The light emitted into non-paraxial modes exhibits a complex pattern because of the atomic disorder in the disks. The same parameters as for Fig. 2 have been used.

4. Photonic band gaps

Let us now turn to the study of band gaps. The lattice reflectivity $R = |r_{N_d}|^2$ and the spontaneous emission $SE = 1 - R - T$ (where $T = |t_{N_d}|^2$ is the lattice transmissivity) in the microscopic model are in accord with the predictions of the TM theory (see Fig. 4(a), (b) and Eq. (5)). Here the reflection and transmission