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Comparison between laboratory experiments and coupled simulations of saucer-shaped hydraulic fractures in homogeneous brittle-elastic solids

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ABSTRACT

Hydraulic fractures that grow at shallow depth or, more generally, near a free surface, curve towards the surface to become saucer-shaped. These saucer-shaped hydraulic fractures pose challenges for modeling that include the need to track the evolution of the crack path and to follow two distinct moving boundaries corresponding to the leading edge of the crack and the fluid front. Results from a coupled, implicit time stepping numerical model agree well with detailed laboratory experimental data for fluid-driven cracks in glass and PMMA. Specifically, the model and laboratory results show good agreement for the crack path, the evolution of the fluid and fracture fronts, the crack opening, and the injection fluid pressure. This strong comparison not only demonstrates the viability of the numerical model, but more generally the results demonstrate that considering coupling among fluid flow, elastic deformation, and radially symmetric crack growth captures enough of the relevant physical processes to accurately predict the leading order behavior of the physical system realized in the laboratory using homogeneous brittle-elastic solids.

1. Introduction

Modeling of fluid-driven cracks, often called "hydraulic fractures", has been the subject of numerous investigations since the pioneering work of Khristianovic and Zheltov (1955) (see Adachi et al., 2007 for an extensive list of references, with a particular focus to the Petroleum Industry). The thrust of these efforts has been directed, however, towards the mechanics of deep hydraulic fractures, with subsurface hydraulic fractures receiving comparatively little attention. Nonetheless, there are numerous circumstances when the propagation of a fluid-driven crack in a solid medium is influenced by the presence of a free surface. In particular, a crack that initiates parallel to a free surface starts to curve towards this surface once it attains a size similar to its initial depth, to eventually form a saucer-shaped crack. Such cracks exist across a wide range of scales and applications, as seen in Table 1, where two industrial examples that do not involve hydraulic loading (Indentation Testing and Ion Blistering) have also been listed.

The propagation of hydraulic fractures entails a coupling between flow of a viscous fluid inside the crack and opening of the crack caused by the internal pressure loading. This nonlinear and nonlocal coupling combines with a competition

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Table 1
Examples of natural and industrial saucer-shaped cracks.

Description	Size (m)	Examples
Mafic sills	10^3–10^5	Chevallier and Woodford (1999) Malthe-Sørenssen et al. (2004) Polteau et al. (2008)
Sand injectites	10^2–10^3	Polteau et al. (2008)
Granite quarrying	10^2	Watson and Laney (1906) Watson (1910)
Mining	10^1–10^2	van As and Jeffrey (2000) Jeffrey and Mills (2000)
Remediation	10^0–10^1	Murdoch and Slack (2002)
Excavation	10^{-1}–10^0	Young (1999)
Laboratory	10^{-2}–10^{-1}	Bunger et al. (2004) Bunger et al. (2008) Galland et al. (2009)
Indentation testing	10^{-4}	Lardner et al. (1990)
Ion blistering	10^{-7}–10^{-6}	Höchbauer et al. (1999) Giguère et al. (2005)

between storage of fluid in the crack and leak-off into the formation and a partitioning of the energy dissipation between viscous fluid flow and the creation of new crack surfaces to create a rich response characterized by multiple regimes of propagation that are associated with the existence of multiple time scales (e.g. Detournay, 2004). The interaction of the hydraulic fracture with the free surface brings two additional elements to this complex problem.

The first element is the curving of the crack path towards the free surface once its characteristic dimension L (function of time t) becomes comparable to its depth H, to eventually daylight (Murdoch, 2002; Bunger et al., 2004; Bunger, 2005; Sher and Mikhailov, 2008; Paynter et al., 2006). This curving stems from the geometrical asymmetry of the problem. Indeed, an internally pressurized crack cannot generally propagate under purely tensile (mode I) conditions by remaining parallel to the free surface, as the crack also experiences a shear displacement offset in addition to its opening mode. In fact, the curving of the crack path is essentially controlled by the ratio, denoted χ, of the far-field normal stress σ_ℓ parallel to the free surface to the characteristic stress $K_{Ic}/\sqrt{H}$ (where K_{Ic} is the material toughness) (Zhang et al., 2002). As evidenced by laboratory experiments (Bunger, 2005; Bunger et al., 2004, 2008), the crack path flattens with increasing ratio χ and thus the hydraulic fracture is expected to remain parallel to the free surface beyond $L(t) \gtrsim H$, only when $\chi \gg 1$.

The second element is the presence of a lag between the crack edge and the fluid front. This lag is a direct result of the low stress environment that exists in the vicinity of a free surface, as suggested by the modeling of fluid-driven cracks propagating either far from (Garagash and Detournay, 2000; Garagash, 2006a; Bunger and Detournay, 2008) or close to a free surface (Bunger, 2005; Detournay and Bunger, 2006). Fluid lag has been documented in both laboratory and field observations (Medlin and Masse, 1984; Rubin, 1995; Bunger, 2005). Analysis of the equations shows that the existence of a lag is needed to satisfy both the propagation criterion and the governing coupled elasticity and lubrication equations, when the preexisting compressive stress normal to the crack is small compared to a time-dependent characteristic stress (Garagash, 2006b; Lecampion and Detournay, 2007).

The research reported in this paper builds on two recent complementary efforts: one is a series of near-surface hydraulic fracturing experiments performed in Polymethyl Methacrylate (PMMA) and glass blocks under varying conditions (a selection of data appears in Bunger, 2005; Bunger et al., 2004, 2008); the other is the implementation of a numerical algorithm, OribiC, capable of simulating the growth of a saucer-shaped hydraulic fracture in an elastic half-space (Gordeliy and Detournay, 2011b, in preparation).

On the one hand, the laboratory experiments provide a rich data set that includes the evolution of the inlet fluid pressure and videos of the experiments that can be processed to determine not only the hydraulic fracture footprint and fluid lag at an arbitrary time but also the aperture field through a photometric analysis (Bunger, 2006). The three-dimensional shape of the crack can also be measured directly after the experiments are complete.

On the other hand, OribiC is able to calculate the axisymmetric crack path, the position of the fluid front, the crack aperture and fluid pressure fields as functions of time. It does this by solving a non-linear system of algebraic equations derived from discretizing a set of integro-differential equations deduced from linear elastic fracture mechanics and lubrication theory. It enhances related hydraulic fracturing simulators (e.g. Zhang et al., 2002; Bunger, 2005; Gordeliy and Detournay, 2011b) by incorporating recent developments in the displacement discontinuity method for modeling cracks in elastic solids (Gordeliy and Detournay, 2011a), which makes it possible to efficiently and accurately model the curving of an axisymmetric hydraulic fracture under the influence of the nearby free surface and the far field stresses.

The combined availability of experimental data on saucer-shaped hydraulic fractures and of a numerical algorithm capable, in principle, of simulating the laboratory experiments gives a valuable opportunity to critically assess the classical mathematical model of a hydraulic fracture under the complicating influence of a free surface. Indeed, the basic assumptions on which the model is constructed (linear elastic fracture mechanics and lubrication theory, Newtonian fluid and homogeneous elastic solid) should essentially hold within the context of the experiments. The requirement to match,