Journal of the Mechanics and Physics of Solids 61 (2013) 949–967

Contents lists available at SciVerse ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Stability of anisotropic magnetorheological elastomers in finite deformations: A micromechanical approach

Stephan Rudykh [a,*], Katia Bertoldi [b,c]

[a] Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA
[b] School of Engineering and Applied Sciences, Harvard University, Cambridge, MA, USA
[c] Kavli Institute for Nanobio Science and Technology, Harvard University, Cambridge, MA, USA

ARTICLE INFO

Article history:
Received 4 May 2012
Received in revised form
4 November 2012
Accepted 28 December 2012
Available online 9 January 2013

Keywords:
Magnetorheological elastomer (MRE)
Layered structure
Finite deformations
Instability
Chain-like microstructure

ABSTRACT

We study the stability of magnetorheological elastomers (MREs) undergoing finite deformations in the presence of a magnetic field and derive a general condition for the onset of macroscopic instabilities. In particular, we focus on anisotropic MREs with magnetoactive particles that are aligned along a particular direction, forming chain-like structures. We idealize the microstructure of such anisotropic magnetosensitive elastomers as a multilayered structure and derive an analytical model for the behavior of these materials. The analytical model, together with the derived condition for the onset of instabilities, is used to investigate the influence of magnetomechanical finite deformations on the stability of the anisotropic MREs. While the formulation is developed for generic hyperelastic magnetosensitive elastomers, the results are presented for a special class of soft materials incorporating a neo-Hookean hyperelastic response. The influence of material properties and loading conditions is investigated, providing a detailed picture of the possible failure modes.

1. Introduction

Magnetorheological elastomers (MREs) consist of magnetic particles, such as micron-size iron particles, dispersed in an elastomeric matrix and can undergo large deformations when excited by a magnetic field. It is well known that application of an external magnetic field to MREs results in significant changes in their macroscopic properties (*e.g.*, Jolly et al., 1996; Ginder et al., 2000, 2002; Gong et al., 2005; Varga et al., 2006), so that they have been exploited to design tunable vibration absorbers and damping components (*e.g.*, Ginder et al., 2001; Deng et al., 2006; Lerner and Cunefare, 2008; Hoang et al., 2011), noise barrier system (Farshad and Le Roux, 2004) and sensors (Tian et al., 2011; Zadov et al., 2012).

Recent experiments (*e.g.*, Farshad and Benine, 2004; Danas et al., 2012) revealed that the microstructure of MREs has a strong impact on their macroscopic response. The distribution of the magnetic particles in MREs can be either random (and, consequently, nearly isotropic) or partially aligned by curing in the presence of a magnetic field (see Fig. 1(a) and (b)). The field-induced stiffening has been observed to be significantly increased in anisotropic MREs where the magnetoactive rigid particles are aligned and form chain-like structures (see Fig. 1). In particular, Chen et al. (2007) experimentally observed an increase of the incremental shear modulus for the samples prepared in the presence of higher magnetic fields, and consequently, with more pronounced chain-like microstructures.

* Corresponding author. Tel.: +1 617 253 5087
E-mail address: rudykh@mit.edu (S. Rudykh).

0022-5096/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jmps.2012.12.008

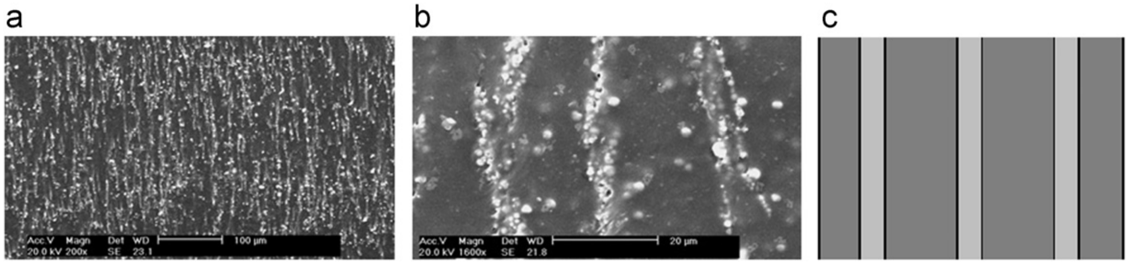

Fig. 1. (a) SEM image with 200 times magnification of MRE prepared in 800 mT (Chen et al., 2007); (b) SEM image with 1600 times magnification of MRE prepared in 800 mT (Chen et al., 2007); (c) schematic representation of the idealized layered microstructure considered in this work. (a) MRE (800 mT) X200. (b) MRE (800 mT) X1600. (c) Idealized MRE.

Theoretical and numerical models have been developed to unravel the mechanics of MREs. Motivated by the development of applications, the pioneering theory of electro and magneto-elastic macroscopic behavior of continuum proposed by Truesdell and Toupin (1960), Tiersten (1964, 1965), and Maugin and Eringen (1972) has been recently reviewed and further developed (Brigadnov and Dorfmann, 2003; Dorfmann and Ogden, 2004, 2005; Bustamante et al., 2006; Bustamante, 2010; Destrade and Ogden, 2011; Han et al., 2012; Thylander et al., 2012). In addition, a homogenization approach has been developed to identify the effective properties of MREs with random distribution of magnetoactive particles (Ponte Castañeda and Galipeau, 2011; Galipeau and Ponte Castañeda, 2012; Galipeau et al., forthcoming).

Since a limiting factor in the design of structures and composite materials is their failure under the applied loads, following the pioneering work of Hill (1957), the investigation of the stability of composites and structures subjected to purely mechanical loadings has attracted considerable attention (Biot, 1965; Hill and Hutchinson, 1975; Triantafyllidis and Maker, 1985; Fleck, 1997; Michel et al., 2007; Bertoldi and Boyce, 2008; Bruno et al., 2010; Rudykh and deBotton, 2012). Moreover, guided by well-established criteria for the "pure" mechanical case, the onset of instabilities for MRE with isotropic distributions of magnetic particles has been investigated focusing on surface instabilities of homogeneous magnetoactive half-space (Otténio et al., 2008) and failure modes of a rectangular MRE block undergoing plane–strain deformation in the presence of a magnetostatic field (Kankanala and Triantafyllidis, 2008).

Here, motivated by recent experiments (*e.g.*, Chen et al., 2007; Guan et al., 2008; Danas et al., 2012), we focus on the stability of MREs with chain-like distributions of magnetic particles and introduce a micromechanical model to describe the behavior of the anisotropic media. To this end, we idealize the material as a multilayered structure (see Fig. 1(c)) and derive an analytical solution for the phase fields. Thus, the dependence of the overall behavior of the material on the volume fractions and material properties of the phases as well as the anisotropy direction is investigated. Next, we derive a general criterion for the onset of the coupled magnetoelastic macroscopic instabilities and further specialize it for the 2D case. By making use of this criterion, the macroscopic stability of multilayered hyperelastic MREs deforming at large strains is systematically investigated and closed form expressions for the identification of unstable domains along different loading paths are determined. We analyze the behavior of the anisotropic MREs in the presence of a magnetic field for three modes of finite deformations: (i) *simple shear in* 2*D*, (ii) *pure shear in* 2*D* and (iii) *axisymmetric shear in* 3*D*. Although the approach is not restricted to a specific choice of constitutive laws of the phases, here we present results for materials with neo-Hookean magnetoactive behavior of the constituents.

2. Theoretical background on magnetorheological elastomers

Let us consider a heterogeneous body and identify with $\mathcal{B}^0$ its undeformed configuration. The application of both mechanical loadings and magnetic fields deforms the body quasistatically from $\mathcal{B}^0$ to the current configuration $\mathcal{B}$. Such deformation is described by the function $\boldsymbol{\chi}$ that maps a reference point $\mathbf{x}^0$ in $\mathcal{B}^0$ to its deformed position $\mathbf{x} = \boldsymbol{\chi}(\mathbf{x}^0)$ in $\mathcal{B}$. The associated deformation gradient will be denoted by $\mathbf{F} = \partial\boldsymbol{\chi}/\partial\mathbf{x}^0$, while J identifies its determinant, $J = \det \mathbf{F}$.

In the absence of mechanical body forces and electric fields, and for deformations applied quasistatically, equilibrium of MREs is ensured when

$$\operatorname{div}\boldsymbol{\sigma} = 0, \quad \operatorname{div}\mathbf{B} = 0 \quad \text{and} \quad \operatorname{curl}\mathbf{H} = 0, \tag{1}$$

where $\boldsymbol{\sigma}$ is the Cauchy total stress tensor, $\mathbf{B}$ is the Eulerian magnetic induction and $\mathbf{H}$ is the Eulerian magnetic field. Moreover, div(•) and curl(•) denote differential operators with respect to $\mathbf{x}$.

Eqs. (1) can be rewritten in terms of the total first Piola–Kirchhoff stress tensor $\mathbf{P} = J\boldsymbol{\sigma}\mathbf{F}^{-T}$, the Lagrangian magnetic field $\mathbf{H^0} = \mathbf{F}^T\mathbf{H}$ and the Lagrangian magnetic induction $\mathbf{B^0} = J\mathbf{F}^{-1}\mathbf{B}$ as

$$\operatorname{Div}\mathbf{P} = 0, \quad \operatorname{Div}\mathbf{B^0} = 0 \quad \text{and} \quad \operatorname{Curl}\mathbf{H^0} = 0, \tag{2}$$

where Div(•) and Curl(•) are the differential operators with respect to $\mathbf{x}^0$.