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Generation of W state and GHZ state of multiple atomic ensembles via a single atom in a nonresonant cavity

Chun-Ling Zhang, Wen-Zhang Li, Mei-Feng Chen *

Laboratory of Quantum Optics, Department of Physics, Fuzhou University, Fuzhou 350002, China

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ABSTRACT

We propose a scheme for generation of the W state and the Greenberger–Horn–Zeilinger (GHZ) state of atomic ensembles. The scheme is based on the dynamics of a single control atom and atomic ensembles interacting with a nonresonant cavity mode. By choosing the appropriate parameters, the effective Hamiltonian describing the interaction between the control atom and the atomic modes shows complete analogy with the Jaynes–Cummings Hamiltonian. The required time for preparing the W state (GHZ state) keeps unchanged (increases linearly) with the increase of the number of atomic ensembles. The effects of dissipation and the detuning between the atomic modes and the control atom on the prepared states are analyzed by numerical simulation.

1. Introduction

Recently, much interest has been paid to the multipartite entanglement. For multipartite systems, there are more peculiar properties than the bipartite ones because they exhibit the contradiction between local hidden variable theories and quantum mechanics even for nonstatistical predictions, as opposed to the statistical ones for the Einstei–Podolsky–Rosen (EPR) states [1]. Moreover, multipartite entanglements are important physical resources for quantum information processing, such as quantum cryptography [2], quantum teleportation [3] and quantum dense coding [4]. The typical multiparticle entangled states are the W state [5] and the GHZ state [5,6], which have been demonstrated to be two inequivalent classes of entangled states. As we know, the W state is robust against qubit loss while the GHZ state is inequivalent to the W state in the sense that it will be reduced to the maximally mixed state when one of the qubits is decohered or to a product state when one of the qubits is measured in the logical basis.

In recent years, many schemes for generation of the W state and the GHZ state have been proposed [7–12]. The physical systems utilized to generate entanglement include superconducting circuits [13–16], linear optical system [17], cavity quantum electrodynamics (QED) [18], trapped ions [19], and quantum dot [20]. Among them, the cavity QED is well developed and regarded as an ideal candidate for quantum communication and quantum state engineering [21,22]. Compared with those schemes that use a single particle as a qubit, the schemes proposed by Lukin et al. [23], Xue and Guo [25], Duan [24], and Han et al. [26] use an atomic ensemble with a large number of identical atoms as the basic system. There are several advantages by using an atomic ensemble as a single qubit. First, the manipulation of the atomic ensemble is normally easier than the coherent control of a single atom for that the laser applied to the atomic ensemble does not separately address the individual atoms in the ensemble [27]. Second, the atomic ensemble that contains a large number of identical atoms increases the light–matter coupling strength, which scales with the square-root of the number of the atoms involved in the ensemble. This greatly reduces the operation time and thus suppresses the decoherence. Those advantages allow one to take a more positive view of the atomic ensemble and regard it as an essential resource for many ingenious applications such as subshot noise spectroscopy and atom interferometry [28], secure cryptography protocols [29], and generation of squeezed states for atomic ensembles [30].

For the generation of entangled states of atomic ensembles, the schemes in Refs. [23–26] are based on single-photon detection, thus the success probabilities of getting the desired states are very small. The scheme in Ref. [25] for preparing the W state and that in Refs. [24,26] for preparing the GHZ state requires the operation time polynomially and exponentially with the number of atomic ensembles. Thus they are also sensitive to the photon loss. Recently, Zheng [31] has found that the dynamics of an atomic system which contains a single control atom and an atomic ensemble can be described as an effective Jaynes–Cummings model (JCM). In his proposal, the atomic ensemble acts as the bosonic mode, and the single control atom and the atomic ensemble are dispersively coupled to a cavity while the control atom is also illuminated by a highly detuned auxiliary

* Corresponding author.
E-mail address: meifchen@126.com (M.-F. Chen).

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classical field. Stimulated by this idea, we present here a new scheme to generate the W state and the GHZ state. The scheme has the following advantages: (i) it does not depend on the photon detection, which simplifies the experimental equipment. (ii) The high-fidelity W state and GHZ state can be achieved even in the presence of the decoherence arising from atomic spontaneous emission and photon leakage. (iii) The required time for preparing the GHZ state increases linearly with the number of atomic ensembles and that for the W state is unchanged

2. Dynamical model of the atomic system

Let us first briefly describe the dynamical model of the system under consideration. A single control atom and an atomic ensemble which contain N identical atoms are trapped in a single-mode cavity. The atomic level configuration and the corresponding transitions are shown in Fig. 1. Each atom has an excited state $|e\rangle$ and two ground states $|f\rangle$ and $|g\rangle$. The atomic transition $|e\rangle \leftrightarrow |g\rangle$ of the control atom (the atoms in the ensemble) is coupled to the cavity with coupling coefficient g_c (g_e) with detuning Δ_g. Meanwhile, the atoms are driven by two classical fields with the Rabi frequencies Ω_1 and Ω_2 and detunings Δ_1 and Δ_2. In the interaction picture, the Hamiltonian describing the system is ($\hbar = 1$)

$$H_I = (\Omega_1 e^{i\Delta_1 t}|e_c\rangle\langle f_c| + \Omega_2 e^{i\Delta_2 t}|f_c\rangle\langle e_c| + g_c e^{i\Delta_g t} a|e_c\rangle\langle g_c|) + \sum_{i=1}^{N}(\Omega_1 e^{i\Delta_1 t}|e_i\rangle\langle f_i| + \Omega_2 e^{i\Delta_2 t}|f_i\rangle\langle e_i| + g_e e^{i\Delta_g t} a|e_i\rangle\langle g_i|) + H.c., \tag{1}$$

where a is the annihilation operator for the cavity mode. Under the large detunings condition, i.e., $\Delta_1, \Delta_2, \Delta_g \gg g_c, g_e, \Omega_1, \Omega_2$, the upper-level $|e\rangle$ can be adiabatically eliminated. Moreover, we set the parameters $\Omega_1 = \Omega_2 = \Omega$ and $\Delta_1 = \Delta_2 = \Delta$ to eliminate the Stark shift induced by the classical pulses. Furthermore, choose the detunings appropriately so that the dominant Raman transition is induced by the classical field Ω_1 and the cavity mode a, while the other Raman transitions are far off-resonant and can be neglected.

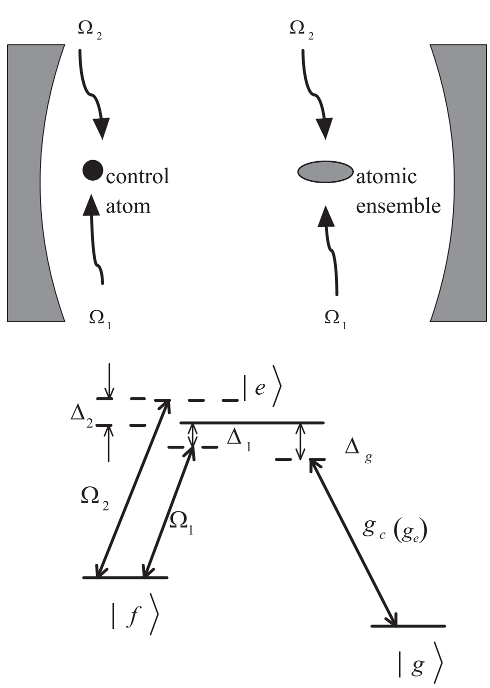

Fig. 1. A single control atom and an atomic ensemble which contain N identical atoms are trapped in a single-mode cavity with different coupling coefficients.

Then the Hamiltonian can be written as

$$H_I' = \frac{g_c^2}{\Delta_g} a^+ a|g_c\rangle\langle g_c| + \sum_{i=1}^{N}\frac{g_e^2}{\Delta_g} a^+ a|g_i\rangle\langle g_i| + (\lambda_c e^{-i\delta t} a|f_c\rangle\langle g_c| + \sum_{i=1}^{N}\lambda_1 e^{-i\delta t} a|f_i\rangle\langle g_i| + H.c.), \tag{2}$$

where $\delta = \Delta_g - \Delta$, $\lambda_c = (g_c\Omega/2)(1/\Delta_g + 1/\Delta)$, and $\lambda_1 = (g_e\Omega/2)(1/\Delta_g + 1/\Delta)$. In the case that $\delta \gg \lambda_1, \lambda_c$, the atoms cannot exchange energy with the field. However, the atoms can exchange energy with each other via the virtual excitation of field mode. Then the Hamiltonian of Eq. (2) can be replaced by the effective Hamiltonian

$$H_{eff} = \frac{\lambda_c^2}{\delta} aa^+|f_c\rangle\langle f_c| + \left(\frac{g_c^2}{\Delta_g} - \frac{\lambda_c^2}{\delta}\right) a^+ a|g_c\rangle\langle g_c| + \sum_{i=1}^{N}\left[\frac{\lambda_1^2}{\delta} aa^+|f_i\rangle\langle f_i| + \left(\frac{g_e^2}{\Delta_g} - \frac{\lambda_1^2}{\delta}\right) a^+ a|g_i\rangle\langle g_i|\right] + \sum_{l=1}^{N}\frac{\lambda_c\lambda_1}{\delta}(S_c^+ S_l^- + S_c^- S_l^+) + \sum_{j,k=1}^{N}\frac{\lambda_1^2}{\delta} S_j^+ S_k^- \quad (j \neq k) \tag{3}$$

where $S_i^+ = |f_i\rangle\langle g_i|$ and $S_i^- = |g_i\rangle\langle f_i|$ $(i = c, 1, 2, \ldots, N)$. Since $[a^+ a, H_{eff}] = 0$, the photon number is conserved during the interaction. If the cavity is initially in the vacuum state, it will remain in this state and the effective Hamiltonian reduces to

$$H_{eff}' = \frac{\lambda_c^2}{\delta}|f_c\rangle\langle f_c| + \frac{\lambda_1^2}{\delta}\sum_{j,k=1}^{N} S_j^+ S_k^- + \frac{\lambda_c\lambda_1}{\delta}\sum_{i=1}^{N}(S_c^+ S_i^- + S_c^- S_i^+). \tag{4}$$

Setting $b^+ = (1/\sqrt{N})\sum_{i=1}^{N} S_i^+$, $b = (1/\sqrt{N})\sum_{i=1}^{N} S_i^-$, $n_b = \sum_{i=1}^{N}|f_i\rangle\langle f_i|$, then we have $[b, b^+] = 1 - (2/N)n_b$. Suppose that the average number of atoms in the state $|f\rangle$ is much smaller than the total atomic number, i.e., $n_b \ll N$, then b and b^+ can be regarded as the bosonic operators. In this case, the Hamiltonian can be rewritten as

$$H_1 = \nu S_{zc} + \varepsilon b^+ b + \mu(S_c^+ b + S_c^- b^+), \tag{5}$$

where $S_{zc} = \frac{1}{2}(|f_c\rangle\langle f_c| - |g_c\rangle\langle g_c|)$, $\nu = \lambda_c^2/\delta$, $\varepsilon = N\lambda_1^2/\delta$, $\mu = \sqrt{N}(\lambda_c\lambda_1)/\delta$, and we have discarded the constant energy λ_c^2/δ. The Hamiltonian H_1 shows complete analogy with the Jaynes–Cummings Hamiltonian. Under the resonant condition

$$\nu = \varepsilon, \tag{6}$$

the Hamiltonian describes the resonant coupling between the control atom and the atomic mode and leads to the transitions

$$|f_c\rangle|0\rangle_e \rightarrow e^{-i(\varepsilon t)/2}[\cos(\mu t)|f_c\rangle|0\rangle_e - i\sin(\mu t)|g_c\rangle|1\rangle_e]; \quad |g_c\rangle|1\rangle_e \rightarrow e^{-i(\varepsilon t)/2}[\cos(\mu t)|g_c\rangle|1\rangle_e - i\sin(\mu t)|f_c\rangle|0\rangle_e], \tag{7}$$

where $|x\rangle_e$ (x=0,1) denotes the Fock state of the atomic ensemble, with x=0 denoting that all the atoms in the ensemble are in the state $|g\rangle$, while x=1 denoting that there is only one atom in the state $|f\rangle$ and the others in the state $|g\rangle$.

In order to validate the feasibility of the above theoretical analysis, we perform a direct numerical simulation of the Schrödinger equation with the full Hamiltonian in Eq. (1) and the effective Hamiltonian in Eq. (5). To satisfy the resonant condition $\nu = \varepsilon$, we set the parameters $g_e = g/\sqrt{N}$ and $g_c = g$. We should mention that the coupling coefficient between the atoms and the cavity mode is dependent on the waist of the cavity and the position of the atoms in the cavity. Hence, the relations $g_e = g/\sqrt{N}$ and $g_c = g$ could be reachable. In the following simulation, we calculate the temporal evolution of the system with the initial state $|f_c\rangle|0\rangle_e$. We plot the time-dependent populations of the basic states $|f_c\rangle|0\rangle_e$ (P1) and $|g_c\rangle|1\rangle_e$ (P2) governed by the full Hamiltonian in Eq. (1) (green lines in Fig. 2) and the effective Hamiltonian in Eq. (5) (red lines in Fig. 2), where $\Omega = g$, $N = 10^4$ and (a) $\Delta = 11g$, $\Delta_g = 12g$; (b) $\Delta = 49g$, $\Delta_g = 50g$. We can see that the effective and full dynamics exhibit excellent agreement when the