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Covariant formulation of the transition radiation energy spectrum of an electron beam at a normal angle of incidence onto a round metallic screen

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ABSTRACT

In the formal expression of the transition radiation (TR) energy spectrum of an *N* electron bunch, the electron spatial coordinates leave the mark of the temporal causality and covariance constraints. In the case of normal incidence, the distribution of the *N* electron longitudinal coordinates determining indeed the temporal sequence of the *N* electron collisions onto the metallic screen rules the emission phases of the *N* single electron radiation field amplitudes from the metallic surface according to a causality constraint. The role of the *N* electron transverse coordinates goes beyond the determination of the relative phase distribution of the *N* electron field amplitudes at the observation point as a function of the transverse displacement of the *N* electrons with respect to the beam axis. The distribution of the transverse coordinates of the *N* electrons being a relativistic invariant under a Lorentz transformation with respect to the direction of motion of the beam is expected to leave a covariant mark on the radiation field amplitudes of the *N* single electrons and, consequently, on both the temporal coherent and incoherent parts of the TR energy spectrum. Compared to the ideal case of a single electron hitting an infinite metallic screen, the covariance of the TR energy spectrum is expected to manifest itself, with the decrease of the beam transverse size, as an asymptotic increase and broadening, respectively, of the radiation spectral intensity and of the corresponding angular distribution. In the case of a round metallic screen with an arbitrary radius, the formal expression of the TR energy spectrum will be derived and numerical results will be presented.

1. Introduction

A relativistic charge crossing a dielectric interface in a rectilinear and uniform motion can originate a highly directional and broad wavelength band radiation emission propagating backward and forward from the boundary surface within a small angle scaling down with the energy of the charge, the so-called transition radiation (TR) [1–9]. Thanks to the instantaneous, highly directional and charge-energy dependent features, TR is a precious tool in the beam diagnostics of a particle accelerator.

TR emitted by a charged beam crossing a thin metallic screen in the vacuum chamber of a particle accelerator can be indeed imaged in the visible by a CCD (Charge-Coupled-Device) camera to monitor the transverse profile of the charged beam or detected in the far-infrared–THz region to determine the bunch length from the spectral analysis of the temporal coherent enhancement of the radiation intensity. Under the keyword TR beam diagnostics, the reader can find an enormous specialized literature dedicated to this topic of which the following reference list [10–39] is a necessarily non-exhaustive bibliography. Under the constraint of a necessarily incomplete bibliography, on the most general subject of the electromagnetic wave interaction, the following references [40–43] can be cited.

For most of the practical detection conditions (from the extreme visible to the THz region), the metallic screen can be assimilated to an ideal conductor surface and the TR emission can be schematized as the result of the dipolar oscillation of the conduction electrons which is induced on the metallic surface by the electromagnetic field of the incident relativistic charge [44]. The dipolar oscillation induced in the double layer of charge by the relativistic charge can indeed explain how an electromagnetic radiative mechanism, originated by a charge in a rectilinear and uniform motion, can also propagate in the backward direction. Such a model of the double layer of charge, describing the TR emission as the result of the dipolar oscillation of the conduction electrons induced by the relativistic charge, is also enlightening about the kinematics of this radiative mechanism and about the common relativistic nature that it shares with other electromagnetic radiative mechanisms by relativistic

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charged beams. The kinematics of the TR mechanism can be indeed schematized as the head-on collision of two distributions of charge as observed in the reference frame of rest of one of the two colliding charged distributions. The backward and forward double conical TR emission can be thus interpreted as the photon bremsstrahlung emission that two head-on colliding electron beams can originate. Taking into consideration the common kinematics and relativistic nature which TR shares with other electromagnetic radiative mechanisms – such as the synchrotron or the bremsstrahlung radiation – it is thus reasonable to expect that, even at a very short wavelength, some spectral modifications of the radiation intensity due to transverse density of the beam should also affect the transition radiative mechanism by an electron beam in a similar way as, in other electromagnetic radiative mechanisms by charged beams, the beam transverse size contributes to determine the so-called Brilliance or Luminosity properties of the radiation source.

In the case of an electron beam at a normal angle of incidence onto a metallic screen with arbitrary size and shape, it can be demonstrated that the covariance and temporal-causality consistent formulation of the TR energy spectrum of an electron beam necessarily implies, even at a very short wavelength, a dependence of the radiation spectral intensity on the distribution function of the particle density in the transverse plane [44,45]. In fact, in the TR emission of an electron beam at a normal angle of incidence onto a metallic screen, the longitudinal and transverse coordinates of the electrons play different roles. The distribution function of the longitudinal coordinates of the N electrons determines indeed the sequence of the particle collisions onto the metallic screen and, consequently, on the basis of the temporal causality principle, also the distribution function of the relative emission phases – from the metallic surface – of the N single electron amplitudes composing the radiation field. The distribution function of the N electron transverse coordinates contributes as well in determining the relative phase delay of the N single electron field amplitudes at the observation point as a function of the transverse displacement of the N electrons with respect to the beam axis where the detector of the radiation field is supposed to be centered. But, in addition, because of the relativistic invariance under a Lorentz transformation in the direction of motion of the beam, the N electron transverse density is expected to leave a covariant mark on the radiation field whose observability can transform but not disappear under a Lorentz transformation [44,45].

In the present work, the covariance and temporal-causality consistent formula of the TR energy spectrum of an N electron bunch, already derived in an implicit form in [44] in the most general case of a radiator surface with an arbitrary size and shape, will be here rendered into an explicit form in the particular case of a round radiator with an arbitrary radius. The so-obtained formula of the TR energy spectrum of an N electron bunch at a normal angle of incidence onto a round metallic screen meets the temporal causality and covariance constraints [44,45] and reproduces, as a limit, some results already well known in the literature, such as the Frank–Ginzburg formula of the single particle TR energy spectrum in the ideal case of an infinite radiator or the single electron TR energy spectrum radiated by a round metallic screen with a finite radius. Finally, because of the covariant dependence of the radiation field on the transverse density of the N electrons, it follows that the distribution function of the N electron transverse coordinates affects both the temporal coherent and incoherent parts of the TR energy spectrum. Even at a very short wavelength, the effect of the N electron transverse density on the spectral distribution of the radiation intensity manifests itself – with the decrease of the beam transverse size – as an increase of the radiated energy and a broadening of the corresponding angular distribution toward the asymptotic limit represented by the ideal case of an electron beam having a point-like transverse extension. In the following, in the case of a bunch of N electrons hitting the metallic screen at a normal angle of incidence, details on the formula of the TR energy spectrum and numerical results will be presented.

2. Transition radiation energy spectrum of an N electron bunch

A bunch of N electrons in a rectilinear and uniform motion along the z-axis of the laboratory reference frame is supposed to collide, at a normal angle of incidence, onto a flat ideal conductor surface S placed in the plane $z=0$. The N electrons are supposed to fly in a vacuum with a common velocity $\vec{w}=(0,0,w)$. All the electrons are supposed to hit the metallic screen at a normal angle of incidence. Effects of the angular divergence of the electron beam on the radiation energy spectrum are not considered in the present work. The radiator surface S has a round shape with a finite radius R. The reference observation point of the radiation emission is supposed to be on the z-axis at a distance D from the screen much larger than the observed wavelength λ. At the time $t=0$ when the center of mass of the charged distribution is supposed to cross the boundary surface, the spatial coordinates of the N electrons are $[\vec{\rho}_{0j}=(x_{0j},y_{0j}),z_{0j}]$ with $j=1,\ldots,N$. The spectral component of the radiation field resulting from the collision of the N relativistic electrons onto the metallic screen reads, see [44–51],

$$E^{tr}_{x,y}(\vec{\kappa},\omega)=\sum_{j=1}^{N}H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})e^{-i(\omega/w)z_{0j}} \tag{1}$$

where, under the far-field approximation [1,2,52], the single electron contribution to the radiation field amplitude $H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})$ can be calculated in the most general case of a radiator surface S with an arbitrary shape and size (either infinite $S=\infty$ or finite $S<\infty$) as follows [44–51]:

$$H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})=\frac{iek}{2\pi^2Dw}\int_S d\vec{\rho}\int d\vec{\tau}\,\frac{\tau_{x,y}\,e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}}{\tau^2+\alpha^2}e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}} \tag{2}$$

where $k=\omega/c=2\pi/\lambda$ is the wave number, $\vec{\kappa}=(k_x,k_y)=k\sin\theta(\cos\phi,\sin\phi)$ is the transverse component of the wave-vector, $\alpha=\omega/w\gamma$ (γ being the relativistic Lorentz factor) and the vector $\vec{\rho}=(x,y)$ represents the integration variable on the surface S of the screen.

With reference to Eqs. (1) and (2), the TR energy spectrum by an N electron beam can be finally calculated as the flux of the Poynting vector, see also [44]:

$$\frac{d^2I}{d\Omega\,d\omega}=\frac{cD^2}{4\pi^2}\sum_{\mu=x,y}\left(\sum_{j=1}^{N}|H_{\mu,j}|^2+\sum_{j,l(j\neq l)=1}^{N}e^{-i(\omega/w)(z_{0j}-z_{0l})}H_{\mu,j}H^*_{\mu,l}\right), \tag{3}$$

where $H_{\mu,j}=H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})$ with $\mu=x,y$, see Eq. (2).

The formal expression of the radiation field and of the radiation energy spectrum of the N electron bunch, as represented in Eqs. (1)–(3) in the most general case of a flat radiator surface S with an arbitrary size and shape, meets the constraints of the temporal causality and of the covariance. The structure of the emission phases from the screen of the N single electron field amplitudes composing the radiation field – see Eqs. (1) and (2) – is indeed in a causality relation to the temporal sequence of the N particle collision onto the metallic screen. Furthermore, with reference to [44,45], covariant or covariance consistent are all the formal steps which, from the expression of the electromagnetic field of the N electron bunch in the laboratory reference frame, lead to the radiation field – see Eqs. (1) and (2) – and to the radiation energy spectrum, see Eq. (3). For more details on the formulation of