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The influence of phase aperture on Beam propagation factor of partially coherent flat-topped beams in a turbulent atmosphere

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ABSTRACT

Analytical formula has been derived for the M^2-factor of a partially coherent flat-topped (PCFT) beam truncated by a phase aperture in turbulent atmosphere based on the extended Huygens–Fresnel principle and the second-order moments of the Wigner distribution function. The M^2-factor of the mentioned beams in atmospheric turbulence have been discussed precisely with numerical analysis. It can be shown that the M^2-factor of a PCFT beam truncated by a phase aperture in turbulent atmosphere increases with propagation distance, and is mainly determined by the parameters of the beam, phase aperture and turbulent atmosphere. The presented results are expected to be useful in long-distance free-space optical communications.

1. Introduction

In recent decades, the propagation of light beams in atmospheric turbulence has been extensively studied [1–3]. In 1990, Siegman proposed the concept of the propagation factor (also known as M^2-factor) [4], which now is an important property of an optical laser beam being regarded as a beam quality factor in many practical applications. The M^2-factor of various laser beams in free space has been studied previously [5–16]. It is found that the M^2-factor of a laser beam remains invariant during propagation in free space. Gori et al. extended the definition of M^2-factor of the partially coherent beam, and studied the M^2-factor of a partially coherent beam in the absence of the aperture [10,11]. Zhang et al. studied the M^2-factor of hard-edge diffracted partially coherent beams [15,16]. Moreover, Zhu et al. considered the M^2-factor of a truncated electromagnetic Gaussian–Schell model beam by using the extended Huygens–Fresnel principle and the second-order moments of the Wigner distribution function (WDF) [12].

There are other studies in which the various aspects have been related to the M^2-factor of main classes of coherent and partially coherent beams [17–24]. In a practical case, most optical systems contain some aperture confinement, and the laser beam propagating through such an optical system usually is truncated, thus it is necessary to evaluate the propagation factor of a truncated laser beam. The propagation properties of some truncated laser beams in free space and turbulent media have been studied in detail recently [15,16,22–26]. To our knowledge no results have been reported up to now about M^2-factor of a partially coherent flat topped (PCFT) beam truncated by a phase aperture. In fact, less attention has been even paid to the propagation of coherent beams through a phase aperture.

A phase aperture can be formed by depositing a dielectric coating through a mask with a single aperture on a substrate of fused quartz [27]. In this paper, our aim is to investigate the properties of the M^2-factor of a PCFT beam truncated by a phase aperture in turbulent atmosphere based on the extended Huygens–Fresnel principle and the second-order moments of the WDF. Analytical expression for the M^2-factor is derived. Also, some numerical analyses are discussed.

2. Analytical formulae for the M^2-factor of a PCFT beam truncated by a phase aperture in turbulent atmosphere

Fig. 1 shows the schematic diagram of a laser beam truncated by a circular phase aperture with phase delay ϕ and radius a is propagating through atmospheric turbulence. In order to describe the M^2-factor properties of a partially coherent beam, a 2×2 cross spectral density (CSD) in the source plane, was introduced [28]

$$W^{(0)}(\vec{r}_1,\vec{r}_2,z=0)=E(\vec{r}_1,z=0)E^*(\vec{r}_2,z=0),\tag{1}$$

where $\vec{r}_1$ and $\vec{r}_2$ are the coordinates of two arbitrary points in the source plane of the beam and "*" denotes the complex conjugate. The CSD of a PCFT beam generated by a Schell-model

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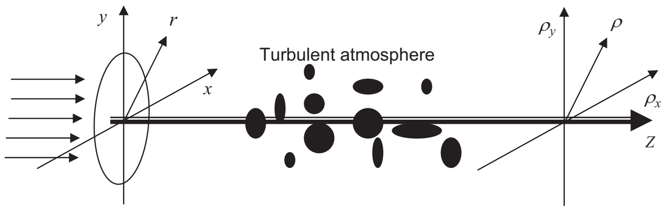

Fig. 1. The schematic diagram of a laser beam truncated by a circular phase aperture in turbulence atmosphere.

source (at $z=0$) can be expressed in the following form [29–37]

$$W^{(0)}(\vec{r}_1,\vec{r}_2,z=0)=\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{(-1)^{n+m}}{N^2}\binom{N}{n}\binom{N}{m}\times\exp\left\{-\left[\frac{n\vec{r}_1^{\,2}}{w_0^2}+\frac{m\vec{r}_2^{\,2}}{w_0^2}+\frac{(\vec{r}_1-\vec{r}_2)^2}{2\sigma_g^2}\right]\right\},\quad(2)$$

where w_0 is the beam waist size of a fundamental Gaussian mode, σ_g is the correlation length, and N is the beam order of a PCFT beam. We assume that a PCFT beam is truncated by a circular phase aperture with radius a at the source plane, so the CSD of the truncated PCFT beam at $z=0$ can be expressed as follows:

$$W_T^{(0)}(\vec{r}_1,\vec{r}_2,z=0)=E(\vec{r}_1,z=0)E^*(\vec{r}_2,z=0)\,T(\vec{r}_1)T^*(\vec{r}_2)\quad(3)$$

with $T(\vec{r}_1)$ as the transmission function of the phase aperture in the form of $T(\vec{r}_1)=e^{-i\varphi}$ for $|\vec{r}_1|\le a$ and $T(\vec{r}_1)=1$ for $|\vec{r}_1|>a$. $T(\vec{r}_1)$ can be expressed in the following alternative form

$$T(\vec{r}_1)=1+[e^{-i\varphi}-1]H(\vec{r}_1)\quad(4)$$

here $H(\vec{r}_1)$ is the transmission function of the circular aperture. $H(\vec{r}_1)=1$ for $\left|\vec{r}_1\right|\le a$ and $H(\vec{r}_1)=0$ for $\left|\vec{r}_1\right|>a$. One might expand $H(\vec{r}_1)$ as the finite sum of complex Gaussian functions [38–41]

$$H\left(\vec{r}_1\right)=\sum_{k=1}^{10}P_k\exp\left(-\frac{G_k}{a^2}r_1^2\right)\quad(5)$$

where P_k and G_k are the expression and Gaussian coefficients, and can be obtained by numerical optimization directly. The table of P_k and G_k can be found in [38]. By Substituting (4) and (5) into (3), we obtain the following expression for the CSD of a PCFT beam truncated by a phase aperture at source plane

$$W_T^{(0)}(\vec{r}_1,\vec{r}_2,z=0)=\sum_{k}^{10}\sum_{l=1}^{10}\sum_{n=1}^{N}\sum_{m=1}^{N}P_kP_l^*\frac{(-1)^{n+m}}{N^2}\binom{N}{n}\binom{N}{m}\times\exp\left\{-\left[\frac{n\vec{r}_1^{\,2}}{w_0^2}+\frac{m\vec{r}_2^{\,2}}{w_0^2}+\frac{(\vec{r}_1-\vec{r}_2)^2}{2\sigma_g^2}\right]\right\}\times\left[1+\left(e^{-i\varphi}-1\right)\exp\left(-\frac{G_k}{a^2}r_1^2\right)\right]\times\left[1+(e^{i\varphi}-1)\exp\left(-\frac{G_l^*}{a^2}r_2^2\right)\right]\quad(6)$$

Let us consider a PCFT beam truncated by a phase aperture propagating in a turbulent atmosphere from the source plane ($z=0$) into the half-space $z>0$. By using the paraxial form of the extended Huygens–Fresnel principle [3], the cross-spectral density function of the mentioned beam through the turbulence can be expressed as [2,3]

$$W_T(\vec{\rho},\vec{\rho}_d,z)=\left(\frac{K}{2\pi z}\right)^2\iint d^2r_1\iint d^2r_2\,W_T^{(0)}(\vec{r},\vec{r}_d,z=0)\times\exp\left[-iK\frac{(\vec{\rho}-\vec{r})^2-(\vec{\rho}_d-\vec{r}_d)^2}{2z}\right]\times\left\langle\exp[\psi(\vec{\rho},\vec{r},z)+\psi^*(\vec{\rho}_d,\vec{r}_d,z)]\right\rangle_m\quad(7)$$

where k is the wave number and the sharp brackets with subscript "m" denote the average over an ensemble of statistical realizations of the turbulent atmosphere. The last term in the integrand of (7) can be written as [3]

$$\left\langle\exp[\psi(\vec{\rho},\vec{r},z)+\psi^*(\vec{\rho}_d,\vec{r}_d,z)]\right\rangle=\exp[-0.5\,D_\psi(\vec{r}-\vec{r}_d)]\cong\exp\left[-\left(\frac{1}{\rho_0^2}\right)\left((\vec{r}-\vec{r}_d)^2+(\vec{r}-\vec{r}_d)(\vec{\rho}-\vec{\rho}_d)+(\vec{\rho}-\vec{\rho}_d)^2\right)\right],\quad(8)$$

with $D_\psi(\vec{r}-\vec{r}_d)$ as the structure function of the refractive index fluctuation and $\rho_0=(0.545C_n^2K^2z)^{-3/5}$ as the coherence length of a spherical wave propagating in the turbulent medium with C_n^2 being the structure constant. In (8) we have used the following summation and difference vector notation

$$\vec{\rho}=\frac{\vec{\rho}_1+\vec{\rho}_2}{2},\quad\vec{\rho}_d=\vec{\rho}_1-\vec{\rho}_2,\quad\vec{r}=\frac{\vec{r}_1+\vec{r}_2}{2},\quad\vec{r}_d=\vec{r}_1-\vec{r}_2\quad(9)$$

where $\vec{\rho}_1$ and $\vec{\rho}_2$ are the coordinates of two arbitrary points in the receiver plane, perpendicular to the direction of propagation of the beam. We express the CSD in the source plane as

$$W_T^{(0)}(\vec{r}_1,\vec{r}_2,z=0)=W_T^{(0)}(\vec{r},\vec{r}_d,z=0)=W_T^{(0)}\left(\vec{r}+\frac{\vec{r}_d}{2},\quad\vec{r}-\frac{\vec{r}_d}{2},\quad z=0\right)\quad(10)$$

It is well known that the WDF is especially suitable for the treatment of PCFT truncated by a phase aperture. The WDF can be shown in terms of the cross-spectral density $W_T(\vec{\rho},\vec{\rho}_d,z)$ as [22,42–44]

$$h(\vec{\rho},\theta,z)=\left(\frac{K}{2\pi}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W(\vec{\rho},\vec{\rho}_d,z)\exp(-iK\theta.\rho_d)d^2\rho_d\quad(11)$$

where vector $\theta=(\theta_x,\theta_y)$, $K\theta_x$ and $K\theta_y$ is the wave vector component along the x-axis and y-axis, respectively. Hence $\theta=(\theta_x^2+\theta_y^2)^{1/2}$ represents an angle of propagation. Integration of function $h(\rho,\theta,z)$ over the angular variables θ_x and θ_y gives the beam intensity, and its integral over the spatial variables ρ_x and ρ_y is proportional to the radiant intensity of the field.

On the basis of inverse Fourier transform of the Dirac delta function and its property of being even function [44], we obtain

$$\delta(\rho'-\rho'')=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\exp[\pm i\kappa_d\times(\rho'-\rho'')]\,d^2\kappa_d\quad(12)$$

Then, the cross-spectral density of the beams in the source plane ($z=0$) can be rewritten as

$$W_T^{(0)}(\vec{r},\vec{r}_d,z=0)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W_T^{(0)}(\vec{r}\,',\vec{r}_d,z=0)\exp[\pm i\kappa_d\times(\vec{r}\,'-\vec{r}_d)]\,d^2\kappa_d d^2r'\quad(13)$$

By substituting (13) into (7) and using (12) in turns out that

$$W_T(\vec{\rho},\vec{\rho}_d,z)=\left(\tfrac{K}{2\pi z}\right)^2\iint d^2\kappa_d\iint d^2r'\,W_T^{(0)}(\vec{r}\,',\rho_d+\tfrac{z}{K}\kappa_d,z=0)\times\exp[-i\rho\times\kappa_d+i\kappa_d\times\vec{r}\,']\times\left\langle\exp[\psi(\vec{\rho},\vec{r}\,',z)+\psi^*(\vec{\rho}_d,\rho_d+\tfrac{z}{K}\kappa_d,z)]\right\rangle_m\quad(14)$$

Inserting (14) into (11) and calculating the integral, the WDF of PCFT beams truncated by a phase aperture through a turbulent atmosphere becomes

$$h(\vec{\rho},\theta,z)=\left(\frac{K}{2\pi}\right)^2\frac{1}{4\pi A_1}A\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}$$