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Phase hologram optimization for the generation of optical fields composed by an arbitrary number of plane waves

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ABSTRACT

We investigate phase holograms (PHs) with optimum diffractive efficiency to generate an arbitrary optical field composed by superposition of an arbitrary number of plane waves. In our approach, we can generate different classes of optical fields since the amplitude and phase of each considered plane wave are independents. In addition, the optical field can be either periodic or nonperiodic. The resulting PH is optimum in the sense that the diffraction efficiency error is minimum. The error is defined as the difference between the upper bound diffraction efficiency and the diffraction efficiency of the proposed PH. Additionally, we guarantee that the optimal solution converges to the global minimum. Illustrative examples and experimental results are presented.

1. Introduction

Complex optical fields have been used in many applications [1–7]. For example, the generation of nonlinear wave guides [1], particle manipulation [2–5], and photonic lattices [6,7]. Phase holograms (PHs) are efficiently used to generate complex optical fields because of their high diffraction efficiency. Furthermore, PHs can be accurately implemented using liquid-crystal spatial light modulator (LC-SLM). Some holographic methods, which encode the desired complex field into phase holograms, have been proposed in the literature [8–10]. In those methods, a high frequency grating distributes the light into different orders of diffraction and one order corresponds to the desired field. In general, a phase hologram $h(x,y)$ can be expressed as [11]

$$h(x,y)=\exp[i\Phi(x,y)]=\beta s(x,y)+\epsilon(x,y), \tag{1}$$

where (x,y) are the cartesian coordinates, $\Phi(x,y)$ is the phase of the PH, β is the amplitude gain, $s(x,y)$ is the desired complex field, and $\epsilon(x,y)$ is the coding noise. $\epsilon(x,y)$ and β depend on the holographic method. Methods [8–10] assume that the overlapping between the spectra of the field and coding noise can be neglected. The amplitude gain for the method [8] is 1, which corresponds to the maximum value of the zero-order Bessel function. Similarly, β equals one, in [9], and is the maximum value of the sinc function. For the method [10], β is equal to 0.582 and corresponds to the maximum value of the first-order Bessel function. The parameter that evaluates the performance of the PH is the diffraction efficiency η given by

$$\eta=\beta^2\eta_0,\quad \eta_0=\frac{\iint_{\Omega_s}|s(x,y)|^2\,dx\,dy}{\iint_{\Omega_s}dx\,dy}. \tag{2}$$

where Ω_s stands for the support that limits the optical field.

In the context of digital holography, the existence of the upper bound diffraction efficiency η_{ub} was first recognized by Wyrowski [11] and η_{ub} is directly related with the upper bound amplitude gain β_{ub} given by [11]

$$\beta_{ub}=\frac{\iint_{\Omega_s}|s(x,y)|\,dx\,dy}{\iint_{\Omega_s}|s(x,y)|^2\,dx\,dy}. \tag{3}$$

The upper bound diffraction efficiency η_{ub} is fulfilled when the phase of the hologram $\Phi(x,y)$ equals the phase of the field $s(x,y)$, i.e., the PH becomes the kinoform of $s(x,y)$ [11,12]. In addition, the Fourier spectra of the field and coding noise do not overlap [13]. It is worth noting that methods [8–10] cannot get the upper bound diffraction efficiency.

Recently, in [12], the authors show that the Fourier spectra of periodic and quasiperiodic nondiffractive optical fields and the corresponding coding noises do not overlap when the kinoform is used to encode the field. Consequently, the desired nondiffractive field can be generated by removing the spectrum of the coding noise in the Fourier domain.

In general, a complex optical field is encoded with upper bound diffraction efficiency using the kinoform. Unfortunately, the spectrum of the encoded field is not always noise free [12]. To illustrate

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this point, consider the following periodic optical field:

$$s(x) = \frac{1}{5} \sum_{n=-2}^{2} \exp[i2\pi n u_0 x], \tag{4}$$

where u_0 is the fundamental frequency in the x direction. Observe that the amplitude and phase for each plane wave are constant. Since the field, kinoform, and coding noise are periodic, the corresponding upper bound amplitude gain and upper bound diffraction are, respectively, $\beta_{ub} = 1.64$ and $\eta_{ub} = 0.53$. The resulting Fourier transform $S(u)$ of $s(x)$ is composed of five Dirac deltas, that is, $S(u) = \frac{1}{5}\sum_{n=-2}^{2} \delta(u - nu_0)$. The amplitudes of Dirac deltas for $S(u)$ are shown in Fig. 1(a). Similarly, Fig. 1(b) and (c) illustrates the amplitudes of Dirac deltas corresponding to the Fourier transforms of the coding noise and kinoform. Fig. 1 suggests that the coding noise cannot be removed in the Fourier domain because the spectra of the field and coding noise are overlapping. The overlapping creates distortions in the encoded field as shown in Fig. 1(c), where the amplitudes of deltas do not possess uniform amplitudes in the interval $-2 \le n \le 2$.

Now, since the methods [8–10] allow us to remove the coding noise in the Fourier domain, we can generate the field $s(x)$ with the following diffraction efficiencies: $\eta = 0.2$ for [8,9] and $\eta = 0.068$ for [10]. However, methods [8–10] do not fulfill the upper bound diffraction efficiency. Therefore, a question that naturally arises in the context of digital holography is how to choose the best PH in terms of the diffraction efficiency for the given complex field.

The present paper studies optimum PH that achieves diffraction efficiency close to the upper bound diffraction efficiency. Particularly, we focus on optical fields composed by superposition of an arbitrary number of plane waves. We also assume that the amplitude and phase of each plane wave are independents. Applications of the proposed method include the generation of sawtooth optical field, which can be used in ratchet systems. The proposed approach is illustrated in both numerically and experimentally.

The rest of the paper is organized as follows. Section 2 focusses on the optimization of the PHs. We describe the codification of complex field composed by superposition of an arbitrary number of plane waves. The amplitude and phase of each plane wave are independent. Depending on the spatial frequencies, the resulting field can be either periodic or nonperiodic. Section 3 provides illustrative examples of the proposed method. Finally, experimental results are given in Section 4.

2. Proposed optimum phase holograms

First, we define the class of optical fields in which we are interested. Consider that the field $s(x, y)$ composed by N plane waves with arbitrary amplitudes and phases obeys the property $|s(x, y)| \le 1$ and is expressed as

$$s(x, y) = \sum_{j=1}^{N} c_j \exp\{[i2\pi(u_j x + v_j y) + i\alpha_j]\}, \tag{5}$$

where the amplitude and phase of the plane wave j are c_j and α_j, respectively, while the spatial frequencies in the x and the y directions are denoted by u_j and v_j. Equivalently, the Fourier transform $S(u, v)$ of $s(x, y)$ consists of N Dirac deltas. To be specific, we have

$$S(u, v) = \sum_{j=1}^{N} c_j \exp(i\alpha_j)\delta(u - u_j)\delta(v - v_j). \tag{6}$$

Next, our goal is to find the best PH for the field $s(x, y)$ given in (5). This means, the PH that minimizes a well-defined objective function. We first consider that the PH is aimed to generate the optical field with desired diffraction efficiency η_{ub}. Accordingly, the PH optimization involves finding the PH that minimizes the diffraction efficiency error $e_\eta = \eta_{ub} - \eta$, where η is the diffraction

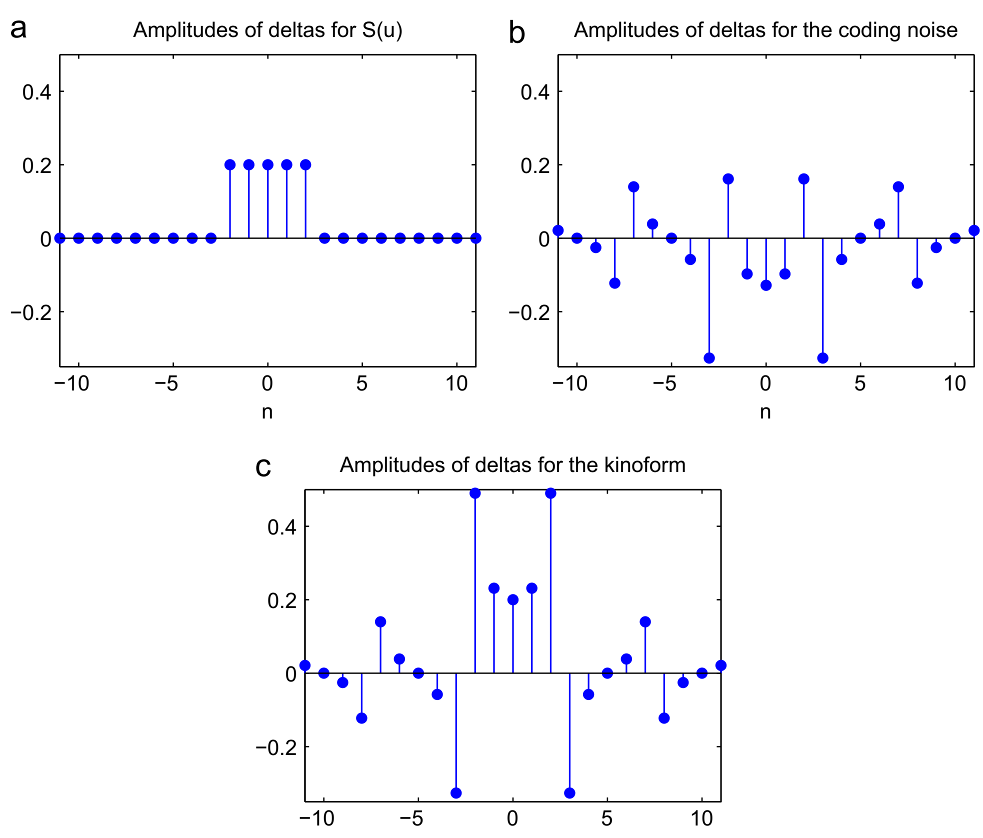

Fig. 1. Amplitudes of the Dirac deltas in the Fourier spectra for (a) the field defined in (4), (b) coding noise and (c) kinoform.