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Scintillation analysis of hypergeometric Gaussian beam via phase screen method

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ABSTRACT

We give a scintillation treatment of hypergeometric Gaussian beams via the use of random phase screens. In particular, we analyse the on-axis, point-like and aperture averaged power scintillation characteristics of this beam that cannot be undertaken easily by analytic means. Within the range of examined source and propagation parameters, our evaluations show that there will be less scintillation, with increasing hollowness at small source sizes and zero topological charge. At larger source sizes or topological charges, this is reversed and decreasing hollowness will reduce scintillation. More or less the same trend is observed for aperture averaging such that at small source sizes and zero topological charge, increased hollowness will result in lower scintillation. At larger source size and topological charges, there will be a transition from the case of smaller values of hollowness giving rise to less scintillation at smaller aperture openings to the case of larger values of hollowness giving rise to less scintillation at larger aperture openings. In general nonzero topological charges will produces more scintillations, both in on-axis and aperture averaged cases.

1. Introduction

Analytic formulation of scintillation induced by turbulent atmosphere requires lengthy derivations, whether Rytov approximation or extended Huygens–Fresnel approach is adopted. This difficulty is alleviated by switching to the random phase screen method, since there finding scintillation is reduced to the simple act of summations and divisions. In the use of random phase screen, there is the added advantage that we can deal with almost any beam type. Finally it is quite easy with random phase screen to extend the single receiver point scintillation calculations to aperture averaged scintillation results, which would again pose enormous difficulties, if it were to be tackled analytically. In this study we have opted for a beam whose scintillation characteristics are difficult to analyse by analytic means and besides have not investigated up to now.

The use of random phase screen to model propagation in turbulence is now well established through quite a number of publications. In two of the earlier works, by adopting a varying power law spectra, the effect of inner and outer scales on intensity variance (scintillation), intensity spectra were analysed, covering strong turbulence regimes for plane and spherical waves [1,2]. For different types of inner scales, the intensity variance results of random phase screen computations and analytic calculations were reported to agree within 2% in [3]. Design considerations of a Kolmogorov phase screen based on the accurate approximation of the phase were discussed in [4]. Errors originating from the use of finite grid size, finite transverse plane dimensions and the separation of phase screens along the propagation path were investigated quantitatively in [5] and expressed for plane and spherical waves. In [6], it was found that simulation of scintillation, beam spreading and reflective speckle patterns via phase screen approach produced results in good agreement with analytic predictions and experimental data. Using a Gaussian beam source, comparisons of random phase screen results with the analytic ones were made in [7] for scintillations, beam spreading, beam wander, coherence diameters, variance and autocorrelation of the beam intensity. Due to the great deal of attention devoted to the subject, two books were published recently covering theoretical backgrounds as well as implementation details of numerical simulation of the optical beam propagation [8,9].

The random phase screen method is also used for scintillation performances of specialized beams. In this context, pseudo-Bessel correlated beams were studied in [10], where it was demonstrated that with the choice of appropriate coherence parameter, these beams could offer lower scintillation both in weak and strong turbulence. In another study [11], the irradiance pattern, degree of polarisation and scintillation index of radially polarised vortex beam represented by the lowest order Laguerre Gaussian beam were investigated and the vector vortex beam was found to posses

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lower scintillation than the scalar counterpart. In [12], again the use of vortex beam of elliptical profile was reported to result in lower scintillation by adjusting the ratio of minor axis to major axis to smaller values. Based on the use of the Gaussian Schell model beam in wave optics simulations, the suitability of gamma and lognormal probability density functions were examined against analytic predictions and it was seen that in weak fluctuation regime, the results of wave optics simulations followed the gamma distribution model, but in strong fluctuations case, the results approximated better to the lognormal probability density function [13]. Scintillation properties of the Gaussian Schell model beam was also investigated in [14] along with the beam spreading and beam wander effects, where it was observed that lower beam coherence would lead to reductions in scintillations and relative beam spreading, but would have less impact on beam wander. As an alternative to the existing random phase screen techniques, a new model named "sparse spectrum" has been proposed and claimed to offer substantial reduction in computation times [15].

It has long been known that as the receiver aperture size is enlarged, aperture averaging will occur and then we will obtain reductions in scintillations [16–20]. But the formulation of aperture averaged scintillation poses extreme difficulties, since it requires the evaluation of covariance function. Up to now, it has been formulated only for plane, spherical and Gaussian beams. In [21], the aperture averaged scintillation for partially coherent Gaussian beam was investigated in relation to correlation time of the source phase variations and the integration time of the detector and it was found that scintillation advantages of partially coherent Gaussian beam could be realized only when the ratio of these two parameters was less than a certain threshold.

The generation, the free space and turbulence propagation characteristics of hypergeometric Gaussian beams were analysed in [22–24]. Hypergeometric Gaussian beam is considered to be a member of the vortex beam family. In [25–27], it was proposed that the vortex feature of such beams could be used as a type of modulation.

In this study, we perform scintillation analysis on hypergeometric Gaussian beams. Here our motivation is to understand the advantages that may be offered by the use of such a beam in optical links. It is well known that scintillation causes a degradation in the probability of error performance of laser communication systems [20]. Therefore any improvement in that direction would be well accepted. The scintillation characteristics of hypergeometric Gaussian beams are explored with this particular aim in mind. The random phase screen method is used, thus allowing both the point-like (on-axis) and aperture averaged scintillations to be evaluated with ease.

2. Description of the random phase screen method

For a source field $u_s(s_x, s_y)$, the Hygens–Fresnel integral delivers the receiver field $u_r(r_x, r_y, L)$ as follows:

$$u_r(r_x, r_y, L) = \frac{-jk\exp(jkL)}{2\pi L}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} ds_x ds_y u_s(s_x, s_y)\exp\left\{\frac{jk}{2L}\left[(r_x-s_x)^2 + (r_y-s_y)^2\right]\right\} \quad (1)$$

where k is the wave number and related to the source wavelength λ as $k=2\pi/\lambda$, $L$is the on- axis receiver plane location, r_x, r_y and s_x, s_y are the transverse coordinates of the source and receiver sides respectively. Eq. (1) can be interpreted as a two dimensional convolution integral and can be written using the convolution operator $\otimes$, as shown below:

$$u_r(r_x, r_y, L) = u_s(s_x, s_y)\otimes h(r_x, r_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} ds_x ds_y u_s(s_x, s_y) h(r_x-s_x, r_y-s_y) \quad (2)$$

where $h(r_x-s_x, r_y-s_y)$ denotes the collection of terms from Eq. (1) such that

$$h(r_x-s_x, r_y-s_y) = \frac{-jk\exp(jkL)}{2\pi L}\exp\left\{\frac{jk}{2L}\left[(r_x-s_x)^2 + (r_y-s_y)^2\right]\right\} \quad (3)$$

The relationship established on the left hand side of Eq. (2) can also be converted into

$$\begin{aligned} u_r(r_x, r_y, L) &= \mathbf{F}^{-1}\{\mathbf{F}[u_s(s_x, s_y)]\mathbf{F}[h(r_x, r_y)]\} \\ &= \mathbf{F}^{-1}\{\mathbf{F}[u_s(s_x, s_y)]H(f_x, f_y)\} \end{aligned} \quad (4)$$

where $\mathbf{F}$ is the Fourier transform operator and

$$\begin{aligned} H\left(f_x, f_y\right) = \mathbf{F}\left[h(r_x, r_y)\right] &= \exp(jkL)\exp\left[-\frac{2j\pi^2 L}{k}\left(f_x^2 + f_y^2\right)\right] \\ &= \exp(jkL)\exp\left[-j\pi\lambda L\left(f_x^2 + f_y^2\right)\right] \end{aligned} \quad (5)$$

From Eq. (4), we conclude that the receiver field can be found by taking the Fourier transform of source field $u_s(s_x, s_y)$, then multiplying this by the Fourier transform of the transfer function $h(r_x, r_y)$, eventually inverse transforming this product. So all together, this is a three step operation. Although Eq. (2) demonstrates that the same could be achieved in one step, i.e., with a single convolution operation, in practical computation, it is seen that for the level of resolution required, the three step Fourier transform method, based on FFT functions available in most software packages, is much faster than the one step convolution implementation.

It is known that a propagating beam will expand due to diffractive effects. To compensate for this, a scaling factor expressing the ratio between receiver and source transverse plane coordinates must be embedded into Eq. (3) [8].

The above development covers propagation in free space, i.e., a medium free of turbulence. To include the effects of turbulence, the entire propagation length of L is split up into N shorter intervals of $\Delta L = L/N$ and a thin phase screen plane is placed at each interval. These phase screen planes are then used to create random phase distribution over the transverse plane. In this manner, the field at the nth plane is linked to the one at the $(n-1)$th plane as

$$u_r(r_x, r_y, n\Delta L) = \mathbf{F}^{-1}\left(\mathbf{F}\{u_s[r_x, r_y, (n-1)\Delta L]\exp[j\phi(r_x, r_y)]\}H\left(f_x, f_y\right)\right) \quad (6)$$

The random phase distribution $\phi(r_x, r_y)$ is retrieved from one of the phase power spectral density functions adopted. In this study, we have selected von-Karman spectrum, thus the phase power spectral density function will be

$$\Phi\left(f_x, f_y\right) = 0.0036k^2 L C_n^2 L_0^{11/3}\frac{\exp\left[-1.1265 l_0^2\left(f_x^2 + f_y^2\right)\right]}{\left[L_0^2\left(f_x^2 + f_y^2\right) + 1\right]^{11/6}} \quad (7)$$

where L_0, l_0 denote the outer and inner scales of turbulence, C_n^2 is the refractive index structure constant. With the utilisation of Eq. (7), the field given in Eq. (6) will incorporate the turbulence induced effects of this particular spectrum. Here the choice of von-Karman spectrum is justifiable, since we cannot go to the limit of zero inner scale due to finite grid spacing used [7].