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Design of an off-axis helmet-mounted display with freeform surface described by radial basis functions

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ABSTRACT

An off-axis optical system with a freeform surface described by Gaussian radial basis functions for a see-through wide field of view helmet-mounted display (HMD) is presented. By using a freeform surface, an off-axis see-through HMD system with one tilted freeform combiner and four relay optical lenses is designed. An off-axis see-through wide field of view HMD system with a 100 mm eye relief, 15 mm pupil, 45° × 32° FOV, and 60° combiner tilt angle is achieved. For the purpose of comparison, two other off-axis see-through HMDs which have the same specifications, but different surface type of combiner and different use count of relay lenses are designed too. One of the two contributions in this paper is the application of radial basis functions to describe optical freeform surface in a wide field of view off-axis helmet-mounted displays, and the other is a way used to determine a starting point of optimization quickly while designing.

1. Introduction

The official history of helmet-mounted display (HMD) starts almost a century ago, with Albert Bacon Pratt, of Lyndon, Vermont [1], and the development of HMD opens with this pioneering exploration and experiment. Design of a HMD involves many principles such as optical engineering, optical material, optical coating, electronics, manufacturing technology, ergonomics, etc. HMD has come a long way from its origin, but more compact structure and lighter weight are still the goal of designers. To achieve the above mentioned deign target, researchers have made unremitting efforts. Besides, there have been many advances in some ways—light source, optical design and manufacturing, and so on. With incessant requirements for high performance, more and more new techniques and new components have been applied to the design of HMD. Many optical systems for see-through HMD have been reported in the past few years.

Rolland designed a 60° field of view optical see-through head-mounted display using off-axis configuration [2]. A breakthrough in the weight reduction challenge was the work of Chen who developed a helmet visor display using diffractive optical elements (DOE) [3]. BAE Systems has exploited Holographic Optical Waveguide technology in Q-Sight^{TM} family of scalable Helmet-Mounted Displays [4]. With the advent of diamond turning technology which can manufacture polygon and optical freeform surface, people begin to research new mathematical descriptions of freeform surface. Examples of freeform surface descriptions include x–y polynomials [5], Zernike polynomials [6], and φ-polynomials [7]. There is a new way to describe freeform surfaces with radial basis functions (RBF), a meshless surface description first applied to optical system design by Cakmakci et al.

Cakmakci et al. in 2008, proposed and implemented a local optical surface representation as a sum of a linear combination of basic functions. As a design example, a single surface off-axis magnifier with a >15 mm eye relief, 3 mm pupil, and 24° diagonal full field of view was designed [8]. Furthermore, they gave another report that the radial basis function was used to describe a freeform mirror in a dual-element off-axis magnifier with a12 mm exit pupil, 15.5 mm eye clearance, and 20° diagonal full field of view in the same year [9]. Improvement of theory and promotion of application in this field will benefit a lot for optical engineering. So, we design a HMD with large field of view, large pupil size, and long exit pupil relief through the use of freeform surface described by Gaussian radial basis functions.

In this paper, an off-axis see-through helmet-mounted displays that is made up of tilted combiner with a radial basis functions surface representation is achieved. Two other combiners that are described with a 10th order asphere and a 25th order Zernike

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polynomial are used for comparison. Each system under comparison has a 100 mm eye relief, 15 mm pupil, a 45° × 32° FOV, and a 60° combiner tilt angle.

2. Optical freeform surface representation with radial basis functions

A radial basis function [10] is any function that has a radial symmetry and typically takes the form:

$$z(\mathbf{x})=\sum_{i=1}^{N}w_i\phi\|(\mathbf{x}-\mathbf{C}_i)\|, \tag{1}$$

Where w_i represents a coefficient in R to be determined, ϕ is a radial basis function whose form is to be selected, the distance matrix $\|\mathbf{x}-\mathbf{C_i}\|$ presents a choice on the locations and the spatial distributions of both the datasites and the basis centers; $\mathbf{C_i}$ represents a point, or a "center" in R^T whose position is to be determined, and $\|.\|$ represents the traditional Euclidean norm; there are N "centers" (N must also be determined). In some cases this form is augmented by adding a sum of polynomial terms. We approximate an optical surface Z by taking a linear combination of basis functions added to a base conic as

$$Z=\frac{cr^2}{1+\sqrt{1-(1+k)c^2r^2}}+\sum_{i=1}^{N}w_i\phi(\|\mathbf{x}-\mathbf{C_i}\|). \tag{2}$$

Radial basis functions have typically taken one of the following forms: linear, cubic, the thin-plate spline, Gaussian, multiquadric and inverse multiquadric. The Gaussian function has the following advantages that are: simple mathematical form, radial symmetry, smoothness and good analyticity of solution. In this paper, we designed the helmet-mounted display with Gaussian. Let's suppose that the number of input samples which are represented by $\mathbf{C_i}$ is N and the corresponding targets in output space is $\mathbf{d}_N$

$$\begin{aligned}&\sum_{i=1}^{N}w_1\phi\|(x_1-\mathbf{C}_i)\|=d_1\\&\sum_{i=2}^{N}w_2\phi\|(x_2-\mathbf{C}_i)\|=d_2\\&\quad\vdots\\&\sum_{i=1}^{N}w_N\phi\|(x_N-\mathbf{C}_i)\|=d_N,\end{aligned} \tag{3}$$

then, the above equations can be rewritten as follows:

$$\begin{bmatrix}\phi_{11}&\phi_{12}&\cdots&\phi_{1N}\\\phi_{21}&\phi_{22}&\cdots&\phi_{2N}\\\vdots&\vdots&&\vdots\\\phi_{N1}&\phi_{N2}&\cdots&\phi_{NN}\end{bmatrix}\begin{bmatrix}w_1\\w_2\\\vdots\\w_N\end{bmatrix}=\begin{bmatrix}d_1\\d_2\\\vdots\\d_N\end{bmatrix}, \tag{4}$$

where

$$\phi_{ji}=\phi(||\mathbf{x}_j-\mathbf{C}_i||),(j,i)=1,2\ldots N, \tag{5}$$

let

$$\begin{aligned}&\boldsymbol{d}=[d_1,d_2\ldots d_N]^T,\\&\boldsymbol{W}=[w_1,w_2\ldots w_N]^T,\end{aligned} \tag{6}$$

and

$$\boldsymbol{\Phi}=\{\phi_{ji}|(j,i)=1,2\ldots N\}. \tag{7}$$

Rewrite Eq. (4) as:

$$\boldsymbol{\Phi}\mathbf{W}=\mathbf{d}. \tag{8}$$

A Radial Basis Function Network (RBFN) that is used to represent a freeform optical combiner in this paper consists of three layers, as shown in Fig. 1. The connection weight vectors of the input and output layers are denoted as $\boldsymbol{\mu}$ and $\mathbf{W}$, respectively. The first layer consists of the input nodes, which are the x and y locations along the aperture. The basis functions in the hidden layer produce a localized response to the input stimulus. The output nodes form a weighted linear combination of the basis functions computed by the hidden nodes. The output ϕ_i of the ith hidden node, using the Gaussian kernel function as a basis, is given by

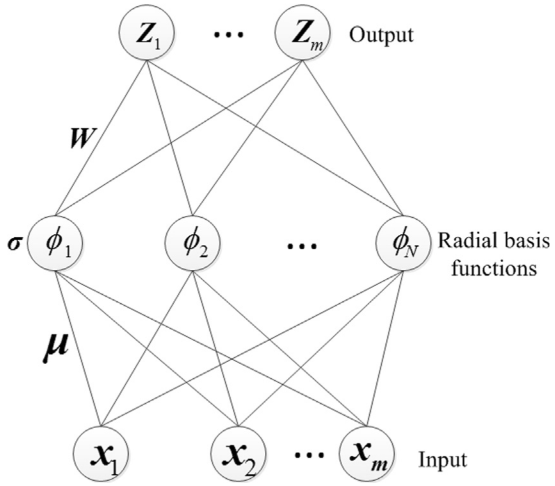

Fig. 1. A Radial Basis Function Network consisting of input nodes, hidden nodes, and output nodes. $\boldsymbol{\mu}$ is stored in the links from the input to hidden layer. σ is the normalization parameter vector of the hidden node activation functions. $\mathbf{W}$ represents the weights of links from the hidden to output layer.

$$\phi_i=\exp\left[-\frac{(\mathbf{x}-\boldsymbol{\mu}_i)^T(\mathbf{x}-\boldsymbol{\mu}_i)}{2\sigma_i^2}\right],\ i=1,2\ldots N, \tag{9}$$

Where $\mathbf{x}$ is the input pattern, $\boldsymbol{\mu}_i$ is its input weight vector (i.e. the center of the Gaussian for node i) and σ_i^2 is the normalization parameter, such that $0\le\phi_i\le1$ (the closer the input is to the center of the Gaussian, the larger the response of the node).

According to Fig. 1, the output $\mathbf{Z}$ is the resulting surface which can be represented in matrix form as Eq. (8):

$$\boldsymbol{\Phi}\mathbf{W}=\mathbf{Z}. \tag{10}$$

3. Design of an off-axis see-through wide field of view HMD optical system

3.1. Display system specifictions

We designed an off-axis see-through wide field of view HMD which comprises an image source, a relay group made of optical elements transparent to the display wavelength, and a catadioptric combiner that was described with the Gaussian radial basis functions. A diagonal 0.6″ (1.55 cm) Emagin Organic Light Emitting Diode (OLED) was selected as the image source, with a resolution of 800 × 600 pixels and a 15 μm pixel size. It yields a Nyquist frequency of 33 cycles/mm. A large exit pupil is important for a flight HMD, so the user will not lose the image if the HMD shifts on his head. A value of 15 mm has been deemed to be an acceptable value for these applications. The eye relief is an obvious characteristic of our design, using a value of 100 mm. This distance is sufficient to allow use of corrective spectacles, nuclear, biological and chemical (NBC) protective masks, and oxygen mask, as well as, to accommodate the wide variations in head and facial anthropometry. Furthermore, the HMD was optimized for a rectangular FOV of 45° × 32° and the bend angle used to fold the light path back to the relay lens and the image source was chosen to be 60°. Specifications of the optical system are listed in Table 1.