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Improved self-imaging for multi-mode optical fiber involving cladding refractive index

Jingjing Zheng [a,b,*], Jing Li [a], Tigang Ning [a], Li Pei [a], Shuisheng Jian [a], Yinghong Wen [b]

[a] Key Laboratory of All Optical Network & Advanced Telecommunication Network of the Ministry of Education, Institute of Lightwave Technology, Beijing Jiaotong University, Beijing 100044, China
[b] EMC Research Section, Beijing Jiaotong University, Beijing 100044, China

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ABSTRACT

In this work, improved self-imaging for optical fiber is proposed. For the first time, a precise formula of reimaging distance involving normalized frequency and cladding refractive index is given. By using our proposal, we can estimate the reimaging distance and evaluate the impact of cladding refractive index precisely and effectively. This method can be used to analyze multi-mode fiber interferences, especially no-core fiber interference.

1. Introduction

Recently, multi-mode fiber interferences (MMFI), which consist of a segment of multimode fiber (MMF), especially no-core fiber (NCF), have attracted great interest among researchers for their significant advantages being neat, compact, easy-fabricating, and flexible. A typical MMFI can be used as a bandpass filter [1] and tunable fiber laser [2–4]. Besides, it is also found that MMI can be used in fiber sensor due to its high-sensitivity to external environment. Lots of applications have been reported, i.e. liquid level measurement [5], curvature sensing [6], strain and temperature sensing [7,8], humidity sensing [9] and liquid refractive index (RI) sensing [10,11]. In previous researches, self-imaging was utilized to describe slab waveguide [12], but not optical fiber [1]. In most theories, the cladding RI, especially external RI in NCF, is not included. Then in Ref. [13], a detailed theory regarding that problem has been investigated. However, the cladding RI was not involved in its final expression of reimaging distance. This theory was referred as mode propagation analysis (MPA) method later in [11]. To the best of our knowledge, the primary difficulty of previous theories ignoring the impact of cladding RI can be concluded as: 1) the impact of cladding RI is un-obvious; 2) its variation range was not that wide as other factors such as core diameter, fiber length and wavelength; and 3) it is not easy to present cladding RI in the formula for reimaging distance.

In this work, we propose a novel expression for reimaging distance, which has a similar form with the traditional self-imaging theory. For the first time, cladding RI is included in the formula. Using the improved self-imaging method, MMFI's performance as well as the impact of cladding RI can be estimated specifically and easily.

2. Theory proposal

The MMFI investigated in this work consists of a step-index MMF and two standard SMFs. Its schematic diagram is shown in Fig. 1. Assuming that the three segments are aligned precisely, thus only the LP_{0m} modes in MMF can be excited well by the input SMF.

The propagation constant β_{0m} for LP_{0m} mode is related with U_{0m} by the equation

$$U_{0m}^2 = a^2(k_0^2 n_{co}^2 - \beta_{0m}^2) \quad (1)$$

with

$$k_0 = 2\pi/\lambda \quad (2)$$

* Corresponding author at: Beijing Jiaotong University, Institute of Lightwave Techonology, Key Laboratory of All Optical Network and Advanced Telecommunication Network of the Ministry of Education, Beijing 100044, China.
E-mail address: jjzheng.c@gmail.com (J. Zheng).

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where α and n_{co} represent the radius and RI of the fiber core, and [14,15]

$$U_{0m} \approx \mu_{0m}(V/V+1) \tag{3}$$

where μ_{0m} is the mth order root of Bessel function J_0, V is the normalized frequency defined as

$$V = \frac{2\pi}{\lambda} a \sqrt{n_{co}^2 - n_{cl}^2} \tag{4}$$

where n_{cl} is the RI of the fiber cladding. Thus the relationship between β_{0m} and V can be concluded as

$$\beta_{0m}^2 = k_0^2 n_{co}^2 - \frac{\mu_{0m}^2}{a^2}\left(\frac{V}{V+1}\right)^2 \tag{5}$$

For high-contrast waveguide, $\mu_{0m}^2/a^2(V/(V+1))^2 \ll k_0^2 n_{co}^2$ is satisfied, thus the propagation constant β_{0m} can be deduced to

$$\beta_{0m} \approx \frac{2\pi}{\lambda} n_{co} - \frac{\mu_{0m}^2 \lambda}{4\pi a^2 n_{co}}\left(\frac{V}{V+1}\right)^2 \tag{6}$$

Considering $V/(V+1) \approx 1$, from Eq. (6) we can draw a simple conclusion which is similar to the one used in traditional self-imaging. In this paper, we use the approximation of

$$(1/(V+1))^2 \ll 1/(V+1) \approx 1/V \tag{7}$$

and deduce β_{0m} to a simple form

$$\beta_{0m} \approx \left(\frac{2\pi}{\lambda} n_{co} - \frac{\mu_{0m}^2 \lambda}{4\pi a^2 n_{co}}\right) - \frac{\mu_{0m}^2 \lambda}{2\pi a^2 n_{co}} \frac{1}{V} \tag{8}$$

In Eq. (8), the most important factor $\Delta\beta_{m1} = \beta_1 - \beta_m$ in self-imaging can be written as

$$\Delta\beta_{m1} = \frac{(\mu_{0m}^2 - \mu_{01}^2)\lambda}{4\pi a^2 n_{co}} + \frac{(\mu_{01}^2 - \mu_{0m}^2)\lambda}{2\pi a^2 n_{co}} \frac{1}{V} = \frac{(\mu_{0m}^2 - \mu_{01}^2)\lambda}{4\pi a^2 n_{co}}\left(1 - \frac{2}{V}\right) \tag{9}$$

A group of numerical calculation results for $\Delta\beta_{21}$ and $\Delta\beta_{10.1}$ from the approximate formula in Eq. (9) and optical waveguide theory for scalar modes are shown in Fig. 2. There are only slight differences between these two methods. For instance, the relative error for $\Delta\beta_{21}$ is within 0.15‰, and 2.5‰ for $\Delta\beta_{10.1}$ in this whole 1300–1800 nm range.

Thus the beat length of the two lowest-order mode L_π is

$$L_\pi = \frac{\pi}{\beta_1 - \beta_2} = \frac{4\pi^2 a^2 n_{co}}{(\mu_{02}^2 - \mu_{01}^2)\lambda(1 - \frac{2}{V})} \approx \frac{\pi^2 a^2 n_{co}}{6.17\lambda(1 - \frac{2}{V})} \tag{10}$$

and the propagation constant spacing [12] can be written as

$$\Delta\beta_{m1} = \frac{(\mu_{0m}^2 - \mu_{01}^2)\pi}{(\mu_{02}^2 - \mu_{01}^2)L_\pi} \tag{11}$$

The first 10 $\Delta\beta_{m1}$ are listed in Table 1. It can be concluded as

$$\Delta\beta_{m1} = \frac{(m-1)(2m+1)}{5} \frac{\pi}{L_\pi} \tag{12}$$

According to Eq. (12), $\Delta\beta_{m1} \cdot 5L_\pi$ is even multiples of π for odd m, and odd multiples of π for even m. Thus the self-imaging condition can be fulfilled at

$$Z_{img} = q(10L_\pi) \quad \text{with} \quad q = 0, 1, 2, \cdots \tag{13}$$

From Eqs. (10) and (13), the complete expression of reimaging distance is

$$Z_{img} = q\frac{40\pi^2 a^2 n_{co}}{(\mu_{02}^2 - \mu_{01}^2)\lambda(1 - \frac{2}{V})} \approx q\frac{10\pi^2 a^2 n_{co}}{6.17\lambda(1 - \frac{2}{V})} \tag{14}$$

The comparison of expressions for the first reimaging distance between the traditional and improved self-imaging is listed in Table 2. With $n_r = n_{co}$ and the approximation of $W_e \approx 2a$, the equation of reimaging for slab waveguide becomes the same with that of optical fiber. Then the difference between the improved

Fig. 1. Schematic diagram of the MMFI investigated in this work; (SMF, single mode fiber; MMF, multimode fiber).

Table 1
The first 10 $\Delta\beta_{m1}$ and relevant parameters.

m	μ_{0m}	$\mu_{0m}^2 - \mu_{01}^2$	$\Delta\beta_{m1}/(\pi/L_\pi)$
1	2.4048	–	–
2	5.5201	24.6884	5/5
3	8.6537	69.1035	14/5
4	11.7915	133.2564	27/5
5	14.9309	217.1487	44/5
6	18.0711	320.7816	65/5
7	21.2116	444.1489	90/5
8	24.3525	587.2612	119/5
9	27.4935	750.1095	152/5
10	30.6346	932.6957	189/5

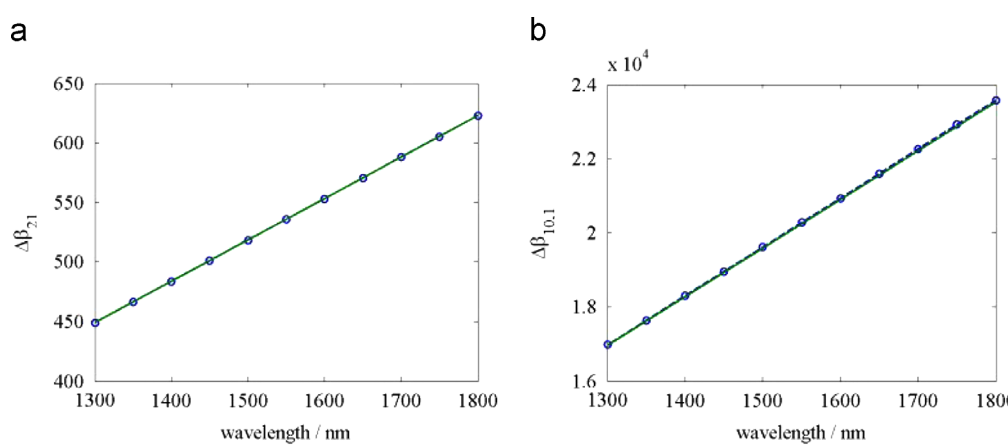

Fig. 2. Numerical calculation results for (1) $\Delta\beta_{21}$ and (2) $\Delta\beta_{10.1}$ from approximate formula Eq. (9) (solid line) and optical waveguide theory for scalar modes (dash line with circles). The pure silicon fiber's diameter is 125 μm and its cladding RI is 1.