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Nonclassical properties of states engineered via coherent operation of photon subtraction and addition on superpositions of coherent states

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ABSTRACT

We investigate nonclassical properties of the optical field when photon subtraction and addition coherent operation $t\hat{a}+r\hat{a}^{+}$ acts on an odd superpositions of coherent states (SCSs) by examining its sub-Poissonian statistics, quadrature squeezing, and negativity of the Wigner function (WF). It is found that these nonclassical properties can be all significantly enhanced for the output state when the coherent operation is applied to an odd SCSs. Moreover, the results show that the nonclassicality of the coherent operation of SCSs (COSCSs) is sensitive to the coherent operation and amplitude.

1. Introduction

Generation and manipulation of the nonclassical light field has been a topic of great interest in quantum optics and quantum information science [1]. Especially at the single-photon level, many theoretical and experimental schemes have been proposed to generate nonclassical states of an optical field. For example, a simple non-Gaussian operation is the photon subtraction, represented by $\hat{a}|\Psi\rangle$ where $\hat{a}$ is the bosonic annihilation operator, which can enhance the performance of the Gaussian two-mode entangled state in quantum teleportation [2–4] and the dense coding [5]. It is further shown that entanglement improvement was experimentally realized by a nonlocal photon subtraction [6] and by the local photon subtractions [7], respectively. Furthermore, it was remarked that entanglement distillation can be achieved also by photon addition, represented by $\hat{a}^{+}|\Psi\rangle$ where $\hat{a}^{+}$ is the bosonic creation operator [8].

On the other hand, some combinations of addition and subtraction, such as photon addition then subtraction and subtraction then addition also were studied. Nevertheless, it is interesting to notice that coherent combinations of two sequences of photon subtraction and addition have been successfully demonstrated experimentally for probing quantum commutation rules by Kim et al. [9,10]. In fact, they have implemented simple alternated sequences of photon creation (addition) and annihilation (subtraction) on a thermal field and observed the noncommutativity of the creation and annihilation operators. In addition, photon subtraction or addition can be applied to improve entanglement between Gaussian states [11,12], loophole-free tests of Bells inequality [13,14], and quantum computing [15].

However, the superposition operation at a more elementary level of first-order field operators, which has attracted much attention by the recent proposal of Lee and Nha, i.e., $t\hat{a}+r\hat{a}^{+}$ was studied with possible applications to quantum-state engineering [16]. Furthermore, it was suggested that some entanglement properties and nonlocality of a two-mode state can also be significantly enhanced by the coherent operation [17,18].

This motivated us to theoretically investigate the coherent operation of photon subtraction and addition from superpositions of coherent states, especially to study its nonclassical properties with respect to the coherent amplitude. To our knowledge, however, this topic has not been previously addressed.

In this paper, we apply the coherent superposition operation $t\hat{a}+r\hat{a}^{+}$ to an odd SCSs and study how the nonclassical properties are effected by the coherent operation on the local mode. This naturally includes the photon subtraction ($r=0$) and the photon addition ($t=0$) as extremal cases. Obviously, the photon subtraction

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operation lets original state an odd SCSs become immediately corresponding to an even SCSs because of $\hat{a}(|\alpha_0\rangle-|-\alpha_0\rangle)=\alpha_0(|\alpha_0\rangle+|-\alpha_0\rangle)$, the photon addition state becomes a single photon exciting odd SCSs. The nonclassical properties are investigated by exploring the sub-Poissonian, quadrature squeezing and the negative WF. It is shown that these nonclassical properties can be all significantly enhanced for the output state when the coherent operation is applied to an odd SCSs. Moreover, the results indicate that the nonclassicality of the COSCSs is sensitive to the coherent operation and amplitude.

The paper is organized as follows. In Section 2, we shall derive the analytical expressions of the normalization constant of the COSCSs. In Section 3, the nonclassical properties is investigated in terms of the sub-Poissonian, quadrature squeezing and the negative WF, it is shown that the coherent operation of photon subtraction and addition can remarkably enhances the nonclassicality of SCSs. The main results are summarized in Section 4.

2. Normalization of the COSCSs

To fully describe a quantum state, its normalization is usually necessary. For this purpose, first, we present two particular types of SCSs, namely, even and odd SCSs, which can be expressed as

$$|SCS_{\pm}(\alpha_0)\rangle=N_{\pm}(|\alpha_0\rangle\pm|-\alpha_0\rangle), \tag{1}$$

where $N_{\pm}=1/\sqrt{2\pm 2e^{-2\alpha_0^2}}$ are the normalization factors and $|\pm\alpha_0\rangle$ is a coherent state of amplitude $\pm\alpha_0$. Here α_0 is assumed to be real for simplicity but without loss of generality. The SCSs with the plus (minus) sign between the coherent states in Eq. (1) is called an even (odd) SCSs because it contains an even (odd) number of photons regardless of the value of α_0. The size of a SCS may be defined by the magnitude of the amplitude α_0.

Note that an odd SCSs in general show stronger nonclassical properties than an even SCSs, consequently, in this paper, we focus our study on effects of photon addition and subtraction on nonclassical properties of the odd SCSs by exploring the sub-Poissonian, quadrature squeezing and the negative WF. After an initial state odd SCSs is operated by the coherent operation $t\hat{a}+r\hat{a}^+$ with $|t|^2+|r|^2=1$, where $\hat{a}$ and $\hat{a}^+$ are creation and annihilation operators, the output state is expressed as

$$|\Psi\rangle=N_-^{-\frac{1}{2}}(t\hat{a}+r\hat{a}^+)(|\alpha_0\rangle-|-\alpha_0\rangle), \tag{2}$$

where N_- are the normalization factors, and the density operator of the resulting state COSCSs is

$$\rho_s=N_-^{-1}(t\hat{a}+r\hat{a}^+)\rho_0(t^*\hat{a}^++r^*\hat{a}), \tag{3}$$

and ρ_0 is the normalized density operator of the SCS

$$\rho_0=N_-^2(|\alpha_0\rangle-|-\alpha_0\rangle)(\langle\alpha_0|-\langle-\alpha_0|), \tag{4}$$

by means of $tr\rho_s=1$, we can obtain the normalization constant for ρ_s

$$N_-=2(|r|^2+\left|t\alpha_0+r\alpha_0^*\right|^2)-2(|r|^2-\left|t\alpha_0-r\alpha_0^*\right|^2)e^{-2|\alpha_0|^2}, \tag{5}$$

3. Nonclassical properties of COSCSs

In this section, we shall discuss the nonclassical properties of COSCSs in terms of sub-Poissonian statistics, quadrature squeezing and negativity of the WF.

3.1. Sub-Poissonian statistics nature of COSCSs

In order to examine sub-Poissonian statistics as the remarkable feature of nonclassical states, we must calculate Mandel's Q-parameter, which is a measure of the sub-Poissonian statistics. Now we study the nonclassical effects of the COSCSs by determining Mandel's Q-parameter, which is defined by

$$Q=\frac{\left\langle\hat{a}^{+2}\hat{a}^2\right\rangle}{\left\langle\hat{a}^+\hat{a}\right\rangle}-\left\langle\hat{a}^+\hat{a}\right\rangle, \tag{6}$$

Using Eq. (2), we can directly calculate:

$$\left\langle\hat{a}^+\hat{a}\right\rangle=N_-^{-1}\left[2(\left|(t+r)\alpha_0^2+r\right|^2+|r\alpha_0|^2)-2(\left|(t-r)\alpha_0^2+r\right|^2-\left|r\alpha_0^2\right|^2)e^{-2\alpha_0^2}\right], \tag{7}$$

$$\left\langle\hat{a}^{+2}\hat{a}^2\right\rangle=N_-^{-1}\left[2(\left|(t+r)\alpha_0^3+2r\alpha_0\right|^2+\left|r\alpha_0^2\right|^2)-2(-\left|(t-r)\alpha_0^3+2r\alpha_0\right|^2+|r\alpha_0|^2)e^{-2\alpha_0^2}\right] \tag{8}$$

Thus the Mandel's Q-parameter can be obtained finally by substituting Eqs. (7) and (8) into Eq. (6). The negativity of the Q refers to sub-Poissonian statistics of the state. However, this is a necessary but not sufficient condition because a state can be nonclassical even of Q is positive, this case is true for the present state.

In order to see clearly the variation of the Q-parameter with α_0, in Fig. 1, we plot Mandel Q factor of the COSCSs as a function of α_0. From which one can clearly see that, for a given α_0 value, the Q parameter becomes negative when r is more than a certain threshold value, and it decreases as r increases. Meanwhile, for a given r value, the Q parameter exists negative only when r is more than a certain threshold value, which increases as α_0 increases; while for a large r, Q is always negative. Especially, nonclassical properties is enhanced remarkably for value of small α_0 and large r, and it is shown that the optimal Q parameter is achieved for $r=1$. This implies that the nonclassicality is enhanced by a coherent operation of addition and subtraction to an odd SCSs, which is shown in Fig. 1.

Furthermore, In order to see clearly the change of the Mandel's Q-parameter after the coherent superposition operation, we plot the Mandel Q factor as a function of α_0 and r upon applying the coherent superposition operation $t\hat{a}+r\hat{a}^+$ for an odd SCSs input with corresponding to the contour plots. The numerical calculation shows that, when $r>0.5859$ the field always obeys sub-Poissonian photon statistics for any value of α_0. Which is shown in Fig. 2. Practically, the same conclusion can also be seen from Fig. 1.

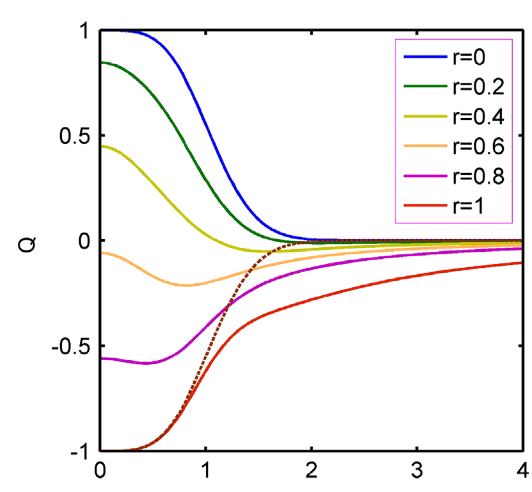

Fig. 1. Mandel Q factor as a function of α_0 upon applying the coherent operation $t\hat{a}+r\hat{a}^+$ for an odd SCSs input with $r=0$, 0.2, 0.4, 0.6, 0.8, 1, respectively, and dashed line represents curve of the odd SCSs.