Optics Communications 309 (2013) 227–230

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Susceptibility for inhomogeneously broadened three-level atomic systems: Simple analytical solutions using Lorentzian distribution

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ARTICLE INFO

Article history:
Received 27 April 2013
Received in revised form
28 June 2013
Accepted 4 July 2013
Available online 18 July 2013

Keywords:
Susceptibility
Doppler effect
Lorentzian

ABSTRACT

We calculate the susceptibility analytically in Doppler-broadened three-level atomic systems using a Lorentzian function instead of a Gaussian function. Based on the results by Noh and Moon [H.R. Noh, H.S. Moon, J. Phys. B 45 (2012) 245002], we obtained simple analytical solutions of the susceptibility using rational functions. We compared the calculated results obtained using a Gaussian function with those obtained using a Lorentzian function, and found good agreement between the two. These analytical results will be useful in various types of laser spectroscopy, such as electromagnetically induced transparency or absorption.

1. Introduction

Since atoms in a vapor cell suffer from inhomogeneous broadening due to the Doppler effect, the susceptibility of a laser field must be averaged over a Doppler-broadened velocity distribution [1]. All the spectra observed in a vapor cell are predicted by averaging the spectra for a single velocity class over a velocity distribution. Many types of spectroscopy employ Doppler averaging, such as Doppler spectroscopy [2,3], Doppler-free spectroscopy [1], coherent population trapping (CPT) [4], electromagnetically induced transparency (EIT) [5,6], and electromagnetically induced absorption (EIA) [7]. The lineshape in EIT is complex so it is usually calculated by a numerical approach. However, analytical solutions are needed when studying the dependence of the linewidths or heights of spectra on various parameters such as the Rabi frequency or decay rates.

The EIT phenomena in a Doppler-broadened medium composed of either ideal three-level atoms [8–11] or real multilevel atoms [12–20] have been studied experimentally and theoretically. The first analytical study of susceptibility was reported by Gea-Banacloche et al. [8]. We introduced a diagrammatic method for calculating the susceptibility in ladder-type three-level atomic systems [10], and applied this method to real ^{87}Rb atoms [21,22]. In a recent paper, we used this diagrammatic method to obtain analytical solutions of susceptibility in Doppler-broadened three-level atomic systems [23,24]. We considered four different schemes, the ladder (upper or lower transition probed), Λ, and V schemes, and obtained general analytic solutions of susceptibility either at equal or unequal wavelengths of the coupling and probe beams. In order to obtain analytical solutions, we assumed that the probe beam was very weak, but that the intensity of the coupling beam was arbitrary.

The analytical solutions, however, were composed of exponential and error functions of complex variables, so it was not easy to manipulate the solutions to other applications. If, however, a Lorentzian function could be employed for velocity averaging instead of a Gaussian function, the analytic results would be more amenable. Since the linewidth of a velocity distribution function is usually much larger than that of a homogeneously broadened spectrum, the velocity distribution function near the resonant condition of the spectrum remains almost constant. Therefore, regardless of the detailed shape of the velocity distribution function, similar spectrum averaged over velocity distribution could be obtained once the central distribution was made to be similar to the Gaussian function. EIT spectra based on a Lorentzian function have been investigated to obtain the analytical form of the EIT linewidth [25–27]. Recently, Tan et al. observed an EIT-ATS (Autler-Townes Splitting) crossover in the Λ type EIT scheme by using a Lorentzian function for velocity averaging [28]. Based on the results in [23], in this paper, we present analytical solutions of susceptibility using a Lorentzian function instead of a Gaussian function. The paper is structured as follows. In Section 2, we describe the method for calculating susceptibility using a Lorentzian function. Analytical solutions based on a Gaussian function are compared with those based on a Lorentzian function in Section 3. Final section summarizes the results of the paper.

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0030-4018/$ - see front matter
http://dx.doi.org/10.1016/j.optcom.2013.07.020

2. Theory

We briefly summarize the method of averaging susceptibility over velocity distribution. Four EIT schemes (Ladder-L, Ladder-U, Λ, and V schemes) are considered. Ladder-L (Ladder-U) denotes the ladder-type EIT scheme where the lower (upper) transition is probed and the upper (lower) transition is driven. Of these, the Ladder-L scheme is shown in Fig. 1(a). In Fig. 1(a), the resonant wavelengths, Rabi frequencies, detunings, and wave vectors of the beams are defined as λ_i, Ω_i, δ_i, and $k_i = 2\pi/\lambda_i$, respectively (i=1, 2). In all schemes except the Ladder-U scheme, i=1 for the probe beam and i=2 for the coupling beam, and *vice versa* for the Ladder-U scheme. The method of calculation of susceptibility is explained with respect to the Ladder-L scheme. The decay rates of the states $|3\rangle$, $|2\rangle$, and $|1\rangle$ are Γ_2, Γ_1, and 0 in the Ladder-L scheme, respectively. γ_{ij} denotes the decay rate of the optical coherence between the states $|i\rangle$ and $|j\rangle$ with i, j=1, 2, 3. Then, the susceptibility of a probe beam can be expressed in terms of optical coherence (ρ_{21}):

$$\chi = -N_{\rm at}\frac{3\lambda_1^3}{4\pi^2}\frac{\Gamma_1}{\Omega_1}\rho_{21}, \quad (1)$$

where $N_{\rm at}$ is the atomic density. When the Doppler effect is taken into account, ρ_{21} becomes a complicated function of velocity v. It is useful to express ρ_{21} in the following form:

$$\rho_{21} = \sum_{j=1}^{N}\frac{b_j}{v-v_j}, \quad (2)$$

where b_j and v_j are complex numbers, and N is an integer. Since we are interested in concise analytical results within the weak probe-beam intensity limit, optical coherence is expressed as in Eq. (2). Otherwise, the density matrix element possesses terms like $(v-v_j)^l$ with $l \neq -1$, which will not be considered here.

Eq. (1) can be averaged over a Maxwell–Boltzmann velocity distribution as

$$\chi_{\rm G} = -\chi_1\sum_{j=1}^{N}\int_{-\infty}^{\infty}dv\frac{e^{-(v/u)^2}}{\sqrt{\pi}u}\frac{b_j}{v-v_j}$$
$$= -\chi_1\sum_{j=1}^{N}\int_{-\infty}^{\infty}dx\frac{e^{-x^2}}{\sqrt{\pi}}\frac{b_j}{x-w_j},$$

where the subscript G implies a Gaussian function, u is the most probable speed, $w_j = v_j/u$, and $\chi_i = N_{\rm at}(3\lambda_i^3/4\pi^2)(\Gamma_i/\Omega_i)$ (i=1, 2). Because the integration is performed as

$$\int_{-\infty}^{\infty}\frac{e^{-x^2}}{x-z} = is\pi W(sz),$$

where $s = {\rm sign}[{\rm Im}(z)]$ and $W(y) = e^{-y^2}\,{\rm Erfc}(-iy)$, the Faddeva function, susceptibility is given by

$$\chi_{\rm G} = -i\sqrt{\pi}\chi_1\sum_{j=1}^{N}b_js_jW(s_jw_j), \quad (3)$$

where $s_j = {\rm sign}[{\rm Im}(w_j)]$.

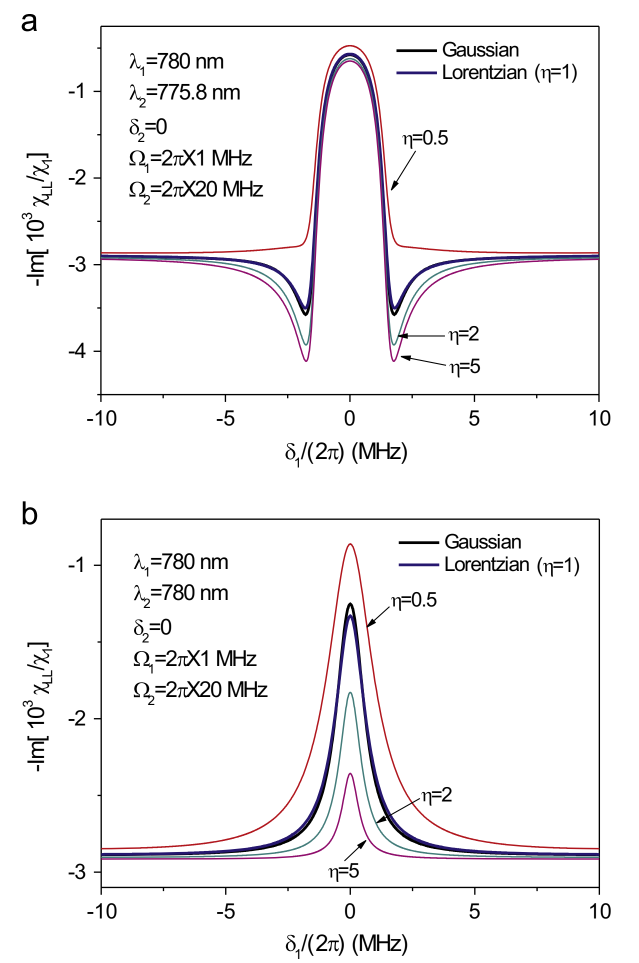

Fig. 2. Imaginary part of susceptibility for Ladder-L scheme. The wavelengths are (a) $\lambda_1 = 780$ nm and $\lambda_2 = 775.8$ nm, and (b) $\lambda_1 = \lambda_2 = 780$ nm. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

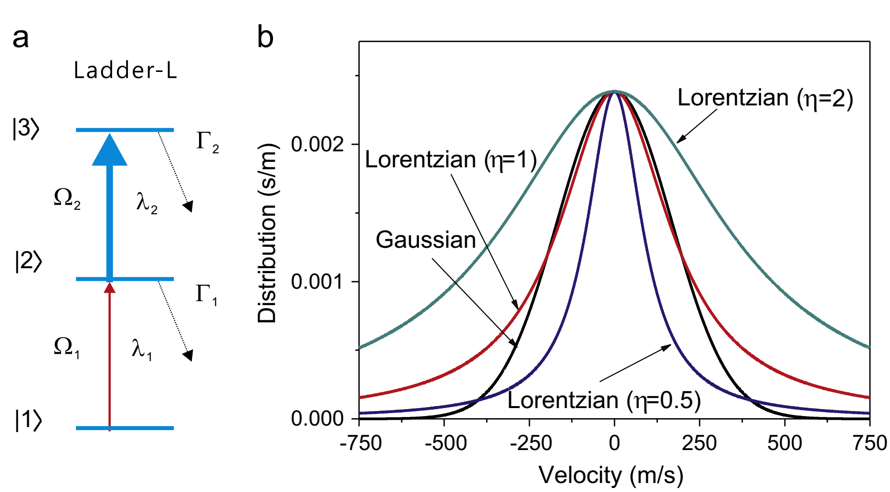

Fig. 1. (a) Energy level diagram for Ladder-L scheme and (b) Gaussian and Lorentzian velocity distribution functions.