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Improved quantum correlations in second harmonic generation with a squeezed pump

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ABSTRACT

We investigate the effects of a squeezed pump on the quantum properties and conversion efficiency of the light produced in single-pass second harmonic generation. Using stochastic integration of the two-mode equations of motion in the positive-P representation, we find that larger violations of continuous-variable harmonic entanglement criteria are available for lesser effective interaction strengths than with a coherent pump. This enhancement of the quantum properties also applies to violations of the Reid–Drummond inequalities used to demonstrate a harmonic version of the Einstein–Podolsky–Rosen paradox. This could offer a real practical advantage over increasing the laser intensity, which will eventually damage the crystal, or using a larger crystal, in which case dispersion problems can be accentuated. We find that the conversion efficiency is largely unchanged except for very low pump intensities and high levels of squeezing.

1. Introduction

One of the simplest non-linear optical processes is travelling wave second harmonic generation [1], in which a nonlinear $\chi^{(2)}$ crystal is pumped with an electromagnetic field at one frequency and produces a second harmonic field at twice this frequency. A comprehensive classical treatment of this process was first given by Armstrong et al. [2]. The quantum properties of the output fields were first calculated using a method of linearisation about the classical solutions [3], even though it had long been known that these were not accurate for arbitrary interaction strengths [4]. The approach of Walls and Tindle, using matrix equations for the number state coefficients, was necessarily limited to small photon numbers. The later development of the positive-P representation [5], which maps the quantum evolution equations onto stochastic differential equations in a doubled phase space, allowed for the treatment of much larger photon numbers. For this system, this was first taken advantage of by Olsen et al. [6] to treat the two-mode model, finding that full conversion to the second harmonic did not occur, but that the fundamental field experienced a revival inside the crystal. This approach enabled a calculation of the quantum properties, such as quadrature squeezing, of the output fields without relying on any assumptions about the mean-field solutions. The positive-P representation was also used to calculate the QND (Quantum Non-Demolition) properties of the system [7] and also compared with results found using the semi-classical theory of stochastic electrodynamics [8]. The quantum correlations between the two fields were also calculated [9], using correlations which later became famous as the Duan–Simon criteria for two-mode continuous-variable entanglement [10,11]. The entanglement between the fundamental and harmonic fields was later named harmonic entanglement by Grosse et al. [12], who calculated it for a system which could operate in both the up and down-conversion regimes. The Reid correlations [13] for Einstein–Podolsky–Rosen entanglement [14] between the two modes have also been calculated previously [15]. In the spirit of Grosse et al., we shall name this harmonic steering. Generally speaking, the production, analysis, and use of these types of correlations using quadratures falls within the area of continuous-variable quantum information [16].

The idea that the conversion efficiency in nonlinear optical processes could be a function of the quantum statistics of the inputs was raised by Shen [17] in 1967. Shen showed that the conversion efficiency in the two-mode model of second harmonic generation would depend on the second-order correlation function, $g^{(2)}(0)$ [18], of the pump, predicting that light with chaotic statistics would initially convert twice as efficiently as a coherent pump. This was later verified by stochastic integration, where the differences in efficiency between these pumps were explicitly calculated [19] without making the small interaction approximations required in Shen's approach. The development of algorithms to model different quantum states in the positive-P and Wigner representations [20] allowed for the investigation of the effects of these in the process of atom–molecule conversion in trapped

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Bose–Einstein condensates [21,22], showing that the initial quantum statistics would also affect this process. Because squeezed states with low average photon number have $g^{(2)}(0)>1$, we decided to investigate the effects of input squeezing in this system. In this work, we use the method given in Ref. [20] to model the input to our crystal as squeezed states with different degrees of amplitude and phase quadrature squeezing, and investigate the outputs in terms of conversion efficiency, single-mode squeezing, two-mode entanglement, and the EPR paradox as expressed by the Reid criteria.

2. Squeezed states

Before we enter into the calculations for the problem, we will review the definition of squeezed states and the quadratures as we define them in this work. This is necessary as there are several different conventions found in the literature, which can lead to confusion if the quantities used are not explicitly defined. We begin with the bosonic annihilation operators, $\hat{a}$ for photons in the fundamental mode at frequency ω, from which we define the quadrature operator

$$\hat{X}_a=\hat{a}+\hat{a}^{\dagger}, \quad \hat{Y}_a=-i(\hat{a}-\hat{a}^{\dagger}), \tag{1}$$

with similar definitions for the harmonic mode at 2ω, using the operator $\hat{b}$. With this definition of the quadratures, the vacuum or coherent state level of the variances is

$$V(\hat{X}_j)=V(\hat{Y}_j)=1, \tag{2}$$

where $j=a,b$.

A squeezed state is then a state of the electromagnetic field that has a variance of less than one in one of the quadratures, at the expense of increased fluctuations in the orthogonal quadrature. We note here that the quadratures can be defined at any angles, but that zero and $\pi/2$ are sufficient for our purposes here. A coherently displaced squeezed state is written as $|\alpha, re^{i\phi}\rangle$, where the c-number α represents the coherent displacement and r is known as the squeezing parameter, with ϕ the angle of the squeezed quadrature. This state has a mean intensity, $N_a=|\alpha|^2+\sinh^2 r$ and quadrature variances for $\phi=0$, $V(\hat{X}_a)=e^{-2r}$ and $V(\hat{Y}_a)=e^{2r}$. For $\phi=\pi/2$, the $\hat{Y}$ quadrature is squeezed and $\hat{X}$ is anti-squeezed. The formal squeezing operator is $S(\epsilon)=\exp(1/2\epsilon^{*}\hat{a}^2-1/2\epsilon\hat{a}^{\dagger 2})$ [23], where $\epsilon=re^{2i\phi}$. As this operator creates and annihilates photons in pairs, we might expect a squeezed field to lead to increased second harmonic conversion efficiency, which we will investigate in what follows.

The second-order correlation function is

$$g^{(2)}(0)=\frac{\langle\hat{a}^{\dagger}\hat{a}^{\dagger}\hat{a}\hat{a}\rangle}{\langle\hat{a}^{\dagger}\hat{a}\rangle^2}, \tag{3}$$

which can be expressed in terms of the number variance as

$$g^{(2)}(0)=1+\frac{V(N)-\langle\hat{a}^{\dagger}\hat{a}\rangle}{\langle\hat{a}^{\dagger}\hat{a}\rangle^2}. \tag{4}$$

We can immediately see that if we have a super-Poissonian field with $V(N)>\langle\hat{a}^{\dagger}\hat{a}\rangle$, $g^{(2)}(0)$ will be greater than unity and we will find photon bunching. On the other hand, $V(N)<\langle\hat{a}^{\dagger}\hat{a}\rangle$ will give an anti-bunched field.

The explicit expressions for the number variances are found as [1]

$$V(N)=|\alpha|^2\exp(-2r)+2\sinh^2 r\cosh^2 r, \quad X \text{ squeezed}$$
$$V(N)=|\alpha|^2\exp(2r)+2\sinh^2 r\cosh^2 r, \quad Y \text{ squeezed}, \tag{5}$$

from which we see that a field squeezed in the $\hat{X}$ quadrature will tend to exhibit anti-bunching while one squeezed in the $\hat{Y}$ quadrature will tend to exhibit bunching. However, calculations show that even for a coherent amplitude of $\alpha=10^2$, the values of $g^{(2)}(0)$ remain close to unity for the range $0\le r\le 2$ which we use in this work. As the coherent amplitude increases, they diverge even less from unity, so that we would not expect a squeezed pump to result in significantly enhanced conversion efficiency except for very weak pumps. We will investigate the effects on the quantum properties of the output fields below, using stochastic integration.

3. Hamiltonian and equations of motion

In this work we will use a simplified description of travelling wave second harmonic generation which does not treat dispersion within the $\chi^{(2)}$ medium and treats the fields as plane waves at fixed frequencies, ω and 2ω. This approach would not be perfectly accurate if we sought to model an actual experiment, in which case we would need to know the dispersion and loss properties of the actual crystal and the beam profile of the input laser. However, in the general case, it is sufficient to show the effects we are looking for as functions of the degree of squeezing of the pump beam. In this sense we are able to establish the optimal correlations which may be aimed for, with the exact accuracy depending on how closely our ideal situation can be approached, without losing the generality of our treatment.

The interaction Hamiltonian is written as

$$\mathcal{H}=i\hbar\frac{\kappa}{2}\left[\hat{a}^{\dagger 2}\hat{b}-\hat{a}^2\hat{b}^{\dagger}\right], \tag{6}$$

where κ is the effective nonlinearity, $\hat{a}$ is the bosonic annihilation operator for photons at frequency ω, and $\hat{b}$ is the bosonic annihilation operator for photons at frequency 2ω, as defined above in Section 2. Following the standard procedures [24], we map this Hamiltonian onto a set of four stochastic differential equations for the evolution of the c-number variables of the positive-P phase-space representation as the fields traverse the nonlinear medium. Proceeding via the von Neumann and Fokker-Planck equations, we find

$$\begin{aligned}
\frac{d\alpha}{dz}&=\kappa\alpha^{+}\beta+\sqrt{\kappa\beta}\,\eta_1(z),\\
\frac{d\alpha^{+}}{dz}&=\kappa\alpha\beta^{+}+\sqrt{\kappa\beta^{+}}\,\eta_2(z),\\
\frac{d\beta}{dz}&=-\frac{\kappa}{2}\alpha^2,\\
\frac{d\beta^{+}}{dz}&=-\frac{\kappa}{2}\alpha^{+2},
\end{aligned} \tag{7}$$

where the $\eta_j(z)$ are real Gaussian noise terms with the properties $\overline{\eta_j(z)}=0$ and $\overline{\eta_j(z)\eta_k(z')}=\delta_{jk}\delta(z-z')$. The independence of the two noise terms means that α and α^{+} are not complex conjugate except on average, which also holds for β and β^{+}. It is this property which allows us to integrate what are equivalent on average to equations of motion for non-commuting operators. Averages over a large number of trajectories of the system of equations above allow us to find normally ordered operator expectation values, with

$$\overline{\alpha^m\alpha^{+n}}\to\langle\hat{a}^{\dagger n}\hat{a}^m\rangle, \tag{8}$$

and similarly for $\hat{b},\hat{b}^{\dagger}$ and β,β^{+}. In practice we integrate these equations in Matlab and average over at least 10^7 trajectories.

4. Squeezing for different input states

We have investigated the outputs for pump fields which are squeezed in either the $\hat{X}$ (amplitude) or $\hat{Y}$ (phase) quadratures, for squeezing parameters, r=0,.5,1. The results are shown in the