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Chaotic unpredictability properties of small network mutually-coupled laser diodes

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ABSTRACT

The chaotic unpredictability properties of mutually-coupled laser diodes (LDs) are investigated numerically. The unpredictability degree is evaluated quantitatively via the permutation entropy (PE). The effects of coupling strength, frequency detuning, feedback strength, as well as time delays are considered. It is shown that, compared with the unidirectional coupling case, two unpredictability-enhanced chaotic signals can be simultaneously obtained for the mutual coupling case, and the parameters regions contributing to unpredictability-enhanced chaos are also broadened. Besides, the PE values for two mutually-coupled LDs are close to each other, with the exact relationship being related to frequency detuning, due to the leader-laggard relationship in terms of injection locking effect. We also consider small network of mutually-coupled LDs, where the effects of connection topologies and frequency detuning are mainly examined. The small network of mutually-coupled LDs can generate several independent unpredictability-enhanced chaotic signals in parallel, which is extremely useful to substantially increase the bit rate and improve the randomness of random number generators based on chaotic LDs.

1. Introduction

Chaotic laser diodes (LDs) have attracted more and more attention due to their potential applications in secure optical communications systems [1–23] as well as random number generators (RNGs) [24–34]. On the one hand, the synchronization properties of both unidirectionally-coupled and mutually-coupled chaotic LDs have been studied extensively [1–14]. Especially, the chaos synchronization and communication among networks of chaotic LDs have attracted lots of attention recently, due to the rapid development of complex system and complex network [15–23]. On the other hand, Uchida et al. [24] demonstrated fast high speed RNGs based on chaotic LDs recently. Since this pioneer work, chaotic LDs have been considered as popular physical entropy sources for high speed RNGs [25–34], and great progress has been made in improving the rate of random bit sequences. Argyris et al. employed a photonic integrated circuit (PIC) that emitted broadband chaotic signals for successful generation of random bit sequences at 140 Gbps [26]. Kanter et al. adopted a high derivative of the digitized chaotic laser intensity and generated the random sequence by retaining a number of the least significant bits of the high derivative value, where an effective output rate of 300 Gbps was achieved [27]. Akizawa et al. proposed to reverse the order of the eight-bit samples of the time delayed signal and perform bitwise XOR operation between the bit-order-reversed samples and the original eight-bit samples, and equivalent generation rate of 400 Gbps was achieved [33]. However, the majority reported techniques of RNGs based on chaotic LDs requires complicated digital post processing to increase the randomness.

In our earlier works, we demonstrated two general photonic approaches to generate unpredictability-enhanced physical chaos, which can further increase the randomness of high speed RNGs [35,36]. It was found that, by introducing dual-path-injection from single master LD, or by adopting dual-chaotic-optical-injection from two different master LDs, the chaotic unpredictability degree of the slave LD could be enhanced significantly compared to the conventional master-slave configuration with single-path-injection, and the parameter regions contributing to unpredictability-enhanced chaos could also be greatly broadened [35,36]. However, in those two systems, only the slave LD can generate unpredictability-enhanced chaos. So far, the chaotic unpredictability properties of mutually-coupled LDs have not been examined, especially when more than two LDs are coupled as small network. Hence, it is interesting and useful to explore whether small

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network of mutually-coupled LDs can simultaneously provide several unpredictability-enhanced chaotic signals in parallel, and thus increase significantly the bit rate of the resulted random numbers.

This work addresses specifically this issue, and the effects of connection topologies on chaotic unpredictability degree of small network of mutually-coupled LDs are also examined. Considering the advantages of simplicity, fast calculation, robustness, we adopt the normalized permutation entropy (PE) to evaluate quantitatively the degree of unpredictability for the chaotic outputs of LDs [37].The remainder of this paper is organized as follows. In Section 2, a theoretical model of mutually-coupled LDs are presented. Besides, the definition of PE is introduced to quantitatively evaluate the unpredictability degree of chaotic signals. In Section 3, for the purpose of comparison, we first consider the case of two mutually-coupled LDs, and compare the unpredictability properties with the unidirectional coupling case. The roles of coupling strength and frequency detuning are mainly examined, and the effects of feedback strength and time delays are also taken into account. Then a system consisting of three mutually-coupled LDs is taken as a basic small network, and the unpredictability properties for different connection topologies are discussed and compared. Finally, conclusions are drawn in Section 4.

2. Theory

The schematic diagrams of the two coupled LDs and three coupled LDs are shown in Fig. 1. Only the fully connected topologies are presented. We introduce connection matrix $A=(a_{mn})$ corresponds to the coupled LDs system, where $m=1,2,3$ and $n=1,2,3$. $a_{mn}=1$ represents the coupling from LDm to LDn, and denotes self feedback for the cases of $m=n.a_{mn}=0$ denotes no coupling from LDm to LDn. For simplicity, we consider homogenous node in the coupled network, and concentrate on the effects of connection between nodes.

2.1. Rate equation model for mutually-coupled LDs

The rate equations for the small network of coupled LDs based on Lang-Kobayashi equations can be read as [38],

$$\frac{dE_m(t)}{dt}=\frac{1}{2}(1+j\alpha)\left(G_m(t)-\frac{1}{\tau_p}\right)E_m(t)+a_{mm}k_{mm}E_m(t-\tau_{mm})e^{-j\omega_m\tau_{mm}} + \sum_{n=1,n\neq m}^{K} a_{nm}k_{nm}E_n(t-\tau_{nm})e^{-j\omega_n\tau_{nm}}e^{j(\omega_n-\omega_m)t} \quad (1)$$

$$\frac{dN_m(t)}{dt}=\frac{I}{q}-\frac{N_m(t)}{\tau_n}-G_m(t)|E_m(t)|^2 \quad (2)$$

$$G_m(t)=g[N_m(t)-N_0]/[1+\varepsilon|E_m(t)|^2] \quad (3)$$

where subscript $m=1,2,3$ stands for LDm. The second term in Eq. (1) denote self feedback and the last term in Eq. (1) represents all coupling into LDm. k_{mm} and τ_{mm} are the feedback strength and feedback delay, k_{nm} and τ_{nm} are the coupling strength and coupling delay, and $\tau_{mm}=\tau_{nm}=3$ns. K is the total number of node in the network. ω_n is the angular frequency of LDn, and we fix $\omega_1=2\pi f_1$ is the angular frequency of LD1 with wavelength at 1550 nm, the frequency detuning between the LDs is defined as $\Delta f_{mn}=(\omega_m-\omega_n)/2\pi=f_m-f_n$. The other parameter description and corresponding values used in the simulations are presented in Table 1. For simplicity, we have neglected noise effects in the LDs. We assume that the internal parameters are identical for all LDs in network. With these parameter values, the relaxation oscillation frequency of the solitary LD is $f_{RO}=5.3$ GHz.

2.2. Quantifier for chaotic unpredictability: Permutation entropy

To illustrate the idea of PE method [37], let us embed a given time series $\{x_t, t=1,\cdots,T\}$ to a d-dimensional space $X_t=[x(t), x(t+\tau_e),\cdots,x(t+(d-1)\tau_e)]$, where $d(\tau_e)$ is the embedding dimension (delay). For practical purpose, Bandt and Pompe suggested using $3\le d\le 7$ with $\tau_e=1$ and indicated that the condition $T>>d!$ should be satisfied to obtain a reliable statistics. The X_t can be arranged as an increasing order of $[x(t+(r_1-1)\tau_e)\le x(t+(r_2-1)\tau_e)\le\cdots\le x(t+(r_d-1)\tau_e)]$, and when $x[t+(r_{t1}-1)\tau_e]=x[t+(r_{t2}-1)\tau_e]$ we order the quantities as $x[t+(r_{t1}-1)\tau_e]\le x[t+(r_{t2}-1)\tau_e]$, if $r_{t1}\le r_{t2}$. Hence, any vector X_t is uniquely mapped onto an "ordinal pattern" $\pi=(r_1,r_2,\ldots,r_d)$. For all the $d!$ possible permutations π of the order d, the probability distribution $P=\{p(\pi)\}$ of the "ordinal patterns" is defined by [37]:

$$p(\pi)=\frac{\#\{t|t\le T-d+1;\ X_t \text{ has type } \pi\}}{T-d+1} \quad (4)$$

Table 1
Parameter sets in the numerical simulation.

Parameter	Description	Value
α	Linewidth enhancement factor	5
τ_p	Photon lifetime	2 ps
τ_n	Carrier lifetime	2 ns
g	Differential gain coefficient	1.5×10^{-8} ps^{-1}
N_0	Carrier transparency	1.5×10^{8}
ε	Gain compression coefficient	5×10^{-7}
I_{th}	Threshold current	14.7 mA
I	Bias current	$1.8I_{th}$
q	Electric charge	1.6×10^{-19}

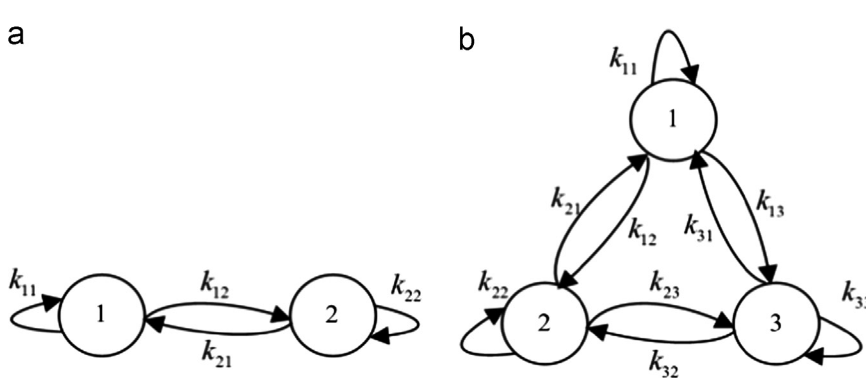

Fig. 1. Schematic diagram of fully connected topologies for (a) two, and (b) three coupled LDs system, 1–3 represent LD1, LD2 and LD3, respectively.

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