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Role of linewidth enhancement factor on the frequency response of the synchronized quantum cascade laser

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ABSTRACT

This paper presents a closed-form analysis of the synchronized quantum cascade laser (QCL) and studies analytically the effect of linewidth enhancement factor (LEF) on the frequency response of the QCL. The analysis predicts that there can be two types of response curves of the QCL—one like series resonant circuit response and the other like antiresonant circuit response. The linewidth enhancement factor introduces asymmetry in the frequency response of the synchronized QCL. The nature of asymmetry in locked QCL is seen to be different from that reported for interband semiconductor lasers. The asymmetry in series resonant-like response is inclined towards the positive detuning side while that in antiresonant circuit-like response is inclined towards the negative detuning side. The degree of asymmetry in QCL response is not only determined by the LEF but also by the upper subband electron relaxation time and the operating wavelength of the QCL. Although the value of LEF in QCL is smaller than that for interband lasers, calculation shows that the LEF has a profound effect on the response of the locked QCL.

1. Introduction

Quantum cascade laser (QCL) [1–23] is being studied as a device of mid-infrared and terahertz coherent source over the wavelength range 3–250 μm for a period of nearly two decades. This is a highly nonlinear device. A quantum cascade laser (QCL) has a series of quantum wells with alternate barrier layers in the active region of the semiconductor in its structure. The active region of this laser is thus designed to have a multi quantum well (MQW) structure surrounded by a thin barrier on one side called the injector and another semiconductor layer on the other side which is known as the collector. The injector injects electrons into the active region and the collector collects the electrons leaving the active region. In QCL, a single conduction band electron falls down a stair-case like potential in the active region of the device generating a series of photons.

The observed linewidth [24] of a semiconductor laser exceeds that predicted by Schawlow–Townes formula and this is attributed to a factor called the linewidth enhancement factor (LEF). The linewidth enhancement factor results from the changes in refractive index and the gain of the active region due to carrier density variation. LEF in QCL although smaller than that for interband semiconductor lasers, plays a dominant role in the nonlinear phenomenon like synchronization of a QCL. A number of papers have been published [25–33] on the measurement of LEF of QCLs.

Due to higher nonlinearity, nonlinear phenomenon like synchronization [14,22,23,34–38] is much more pronounced in QCL. Although much research work is being carried out on the device aspect of QCL, not much work has been done in the circuit aspect of QCL. We have developed a transmission line model for synchronized QCL [34–38]. In this paper, we carry out an analysis of the synchronized QCL taking into account the effect of linewidth enhancement factor (LEF) on the synchronization phenomenon of QCL. The LEF is found to have a profound effect on the frequency response of the synchronized QCL.

2. Calculation of cavity resonance frequency shift due to light injection

We consider a basic unit of a QCL grown with lattice matched AlInAs/GaInAs/InP structure [1,11–13,17] for operation at 4.6 μm and a strain-compensated InGaAs/AlAsSb/InP structure [4,8,9,12,13] for operation at 3.7 μm. At present, room temperature operation of these devices has been possible.

The active region of this unit consists of three GaInAs quantum wells with alternate AlInAs (or AlAsSb) barriers. The electrons have allowed energy levels in the wells known as subbands. There are three subbands in this model. The electrons are injected into the upper most subband of the active region by tunnelling through the injector barrier in a very short time (~ 0.2 ps). Electronic transition takes place from the uppermost subband to the lower-middle

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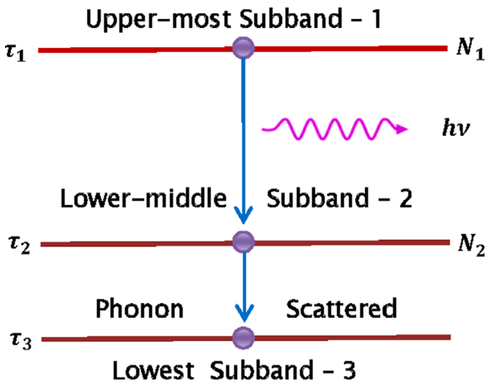

Fig. 1. A three level subband structure of the basic QCL.

subband in the conduction band of the QCL generating a single photon. In a periodic structure, where there exists a series of identical active regions containing barriers and wells, a series of identical photons are emitted due to multiple transitions of an electron in a staircase-like potential of the QCL resulting in high power.

The subband structure of the basic unit of the QCL is shown in Fig.1.

In the three level subband structure, the photon is emitted from the upper-most subband to the lower-middle subband as shown in Fig. 1. From the lower-middle subband, the electrons are removed very fast within a fraction of picosecond (~ 0.6 ps) relaxation time (τ_2) to the lowest subband in a non-rediative manner through optical phonon scattering. Thus, the lower-middle subband is depopulated in a superfast way through non-radiative optical phonon-scattering. The electrons from the lowest subband leave the active region by tunnelling through the barrier in a very short time $\tau_3 \leq 0.5$ ps. These processes help maintain the required population inversion between the upper-most and lower-middle subband levels. The most important difference between normal interband semiconductor laser and the intersubband QCL is that the electron relaxation time (τ_1) in QCL is much smaller (~ 4.3 ps) [1] than the electron lifetime (~ 3 ns) in the internband laser. This three order in magnitude decrease in electron relaxation time makes the QCL an ultrafast device.

The rate Eq. (14) applied to the subband populations in QCL can be written as.

$$\frac{dN_1}{dt} = \frac{I}{eV_s} - \frac{N_1}{\tau_1} - \Gamma GS \tag{1}$$

$$\frac{dN_2}{dt} = \frac{N_1}{\tau_1} + \Gamma GS - \frac{N_2}{\tau_2} \tag{2}$$

where I = bias current of the QCL, V_s = volume of a single active region, e = magnitude of electronic charge, N_1 = electron density in the upper-most subband—1, N_2 = electron density in the lower-middle subband—2, τ_1 = electron relaxation time in the upper-most subband—1, τ_2 = electron relaxation time in the lower-middle subband—2, G = gain of the QCL due to population inversion, S = photon density in the cavity and Γ = optical confinement factor.

In the steady state, $dN_1/dt = 0$ and $dN_2/dt = 0$. With these conditions, adding Eqs. (1) and (2) we get,

$$N_2 = \frac{I\tau_2}{eV_s} \tag{3}$$

The gain of the QCL due to population inversion can be written as

$$G = A_0(N_1 - N_2) \tag{4}$$

where A_0 = differential gain coefficient. Then, Eq. (1) in the steady state yields

$$\frac{N_1}{\tau_1} + A_0(N_1 - N_2)\,\Gamma S = \frac{I}{eV_s} \tag{5}$$

Using Eq. (3) and rearranging Eq. (5), we get

$$N_1 = \frac{I}{eV_s}\tau_1 \frac{1 + \Gamma\tau_2 A_0 S}{1 + \Gamma\tau_1 A_0 S} \tag{6}$$

$$N_1 - N_2 = \frac{I}{eV_s}\frac{\tau_1 - \tau_2}{1 + \Gamma\tau_1 A_0 S} \tag{7}$$

To calculate the change in carrier concentration of the upper-most subband origination from a change in photon density, we take the differentials of N_1 and S in Eq. (5). Subsequent simplification yields

$$\Delta N_1 = -\frac{\Gamma\tau_1 G \Delta S}{1 + \Gamma\tau_1 A_0 S} \tag{8}$$

This is the relation between the change in upper-most subband electron concentration and the change in photon density in the active region.

The change in photon density (ΔS) is related with the change (ΔP) in optical power in the lasing cavity as

$$\Delta P = \frac{h\nu V \Delta S}{(nl/c)} \tag{9}$$

where h = Plank constant, ν = lightwave frequency, n = refractive index of the active region of length l, c = vacuum velocity of light, V = total volume of the active region of the QCL. The cavity resonance frequency changes due to light injection. If $\Delta\omega_0$ = change in resonance frequency in radian of the cavity, then

$$\frac{\Delta\omega_0}{\omega_0} = -\frac{\Gamma}{\bar{n}}\Delta n \tag{10}$$

$$= -\frac{\Gamma}{\bar{n}} k_1 \Delta N_1 \tag{11}$$

where $\bar{n}$ is the group refractive index of the cavity and the change in cavity index, Δn, is equal to $k_1 \Delta N_1$. k_1 is a proportionality constant which relates the refractive index change with the change in carrier density. k_1 is negative since the laser frequency decreases with optical injection. Substituting (8) in (11) we get

$$\frac{\Delta\omega_0}{\omega_0} = +\frac{\Gamma^2 k_1}{\bar{n}}\frac{\tau_1 G \Delta S}{1 + \Gamma\tau_1 A_0 S} \tag{12}$$

Substituting ΔS in terms of ΔP from Eq. (9), Eq. (12) reduces to

$$\frac{\Delta\omega_0}{\omega_0} = \frac{\Gamma^2 k_1}{\bar{n}}\frac{\tau_1 G(\frac{nl}{c})}{1 + \Gamma\tau_1 A_0 S}\frac{1}{h\nu V}\Delta P \tag{13}$$

here G is in s^{-1}, A_0 in $\mathrm{m}^3\,\mathrm{s}^{-1}$, k_1 in m^3, and V in m^3.

If P_i and P_s be the optical injection power and free-running optical power of the slave QCL, and φ be the phase difference between the injection lightwave and the free-running output lightwave of the QCL then

$$\Delta P = 2\sqrt{P_i \times P_s}\cos\varphi \tag{14}$$

assuming $P_i < < P_s$.

The concept of linewidth enhancement factor (LEF) arises from the fact that a change in carrier concentration in the active region of the laser produces a change in gain and also a change in refractive index of the active region. The LEF is directly proportional to the ratio of the rate of refractive index change with the carrier concentration and the rate of gain change with the carrier concentration. The gain (G) of the QCL is directly dependent on the