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Invited Paper

Optimal configuration of partial Mueller matrix polarimeter for measuring the ellipsometric parameters in the presence of Poisson shot noise and Gaussian noise

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ABSTRACT

We address the optimal configuration of a partial Mueller matrix polarimeter used to determine the ellipsometric parameters in the presence of additive Gaussian noise and signal-dependent shot noise. The numerical results show that, for the PSG/PSA consisting of a variable retarder and a fixed polarizer, the detection process immune to these two types of noise can be optimally composed by 121.2° retardation with a pair of azimuths ±71.34° and a 144.48° retardation with a pair of azimuths ±31.56° for four Mueller matrix elements measurement. Compared with the existing configurations, the configuration presented in this paper can effectively decrease the measurement variance and thus statistically improve the measurement precision of the ellipsometric parameters.

1. Introduction

Mueller matrix polarimeter (MMP) can measure all the 16 elements of Mueller matrix of the sample that provide the parameters related to optical properties. As such, MMP has widely used in accurate determination of dielectric function, optical properties and geometric characteristics of thin films [1–8]. A typical MMP consists of a complete polarization state generator (PSG) and a polarization state analyzer (PSA), which can determine the polarization altering properties of a sample both in reflection and in transmission. A PSA/PSG generally consists of a polarizer and an active birefringent optical component either modulated by azimuthal rotation or by an externally applied electric field, such as rotating (waveplate/bi-prism) retarders, electro-optical modulation, photoelastic modulators, and liquid crystal retarders [9–13].

The practical detection process invariably has noises that disturb the measurement and reduce the accuracy of the reconstructed Mueller matrix. Two types of noise are frequently encountered and considered statistically: Gaussian additive noise, representative of sensor noise, and Poisson shot noise that results from the quantum fluctuations of the useful or ambient light flux [14–16]. In the design of MMP, optimizing the instrument matrices of PSG and PSA is an effective way to minimize the estimation variance. The retardations and azimuths of retarders relative to instrument matrices are optimized by using the specified metrics such as condition number (CN) or equally weighted variance (EWV) [17–26]. However, the noise variances in the measured Mueller matrix for such optimized MMPs are still sensitive to the Poisson noise. Anna et al. optimized the instrument matrices by minimizing total variance of all the 16 elements in the measured Mueller matrix for both Gaussian noise and Poisson noise and demonstrated an ideal instrument matrix model [15]. The MMP corresponds to such ideal instrument matrix is insensitive to both Gaussian additive noise and Poisson shot noise. Mu et al. proposed a method that minimizing the Euclidean distance between the practical instrument matrices and the ideal one for determining the optimum configuration of Full-Stokes polarimeter architecture with the immunity to both Poisson and Gaussian noise, which can be also used to determine the optimum configuration of PSG/PSA in MMP [27]. Under certain circumstances, some elements in the Mueller matrix of anisotropic optical materials are zeros that makes the Mueller matrix be block diagonal, the ellipsometric parameters can be determined by only 4 nonzero elements of the Mueller matrix. Thus, the optimal instrument matrix presented by Anna et al. and the relevant configurations determined by Mu et al. could no longer be optimal for the measurement of ellipsometric parameters [28,29]. Li et al. minimized the summation variance of the partial elements in the block diagonal Mueller matrix by using the method proposed by Anna et al. and demonstrated another ideal instrument matrix model [30]. In applications, it is needed to determine practical MMP

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configuration that can generate the ideal instrument matrix for the measurement of block diagonal Mueller matrix used to determine the ellipsometric parameters. However, as far as we know, no such practical configuration has been developed and reported.

In this paper, we address the issue of determining the practical configuration of the MMP for measuring the ellipsometric parameters of anisotropic optical materials with immunity to both Poisson shot noise and additive Gaussian noise. A typical configuration of MMP is optimized by a cost function that accounted for Manhattan distance between the rows of the generated instrument matrices and those of the optimal ones. The performances of our optimized configuration and the existing ones are compared, and the results demonstrate that the configuration proposed in this paper can lead to lower total variance of the Mueller matrix elements related to the ellipsometric parameters.

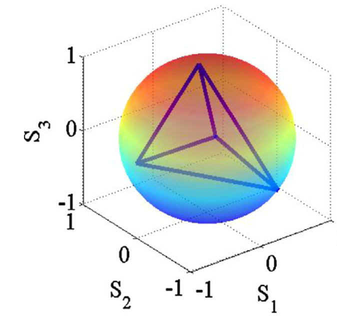

Fig. 1. The last three elements of each row from the 4-elements instrument matrix inscribes a regular tetrahedron with maximum volume inside a Poincaré sphere of unit radius.

2. Theory

2.1. Instrument matrix

The linear equation of MMP is

$$\mathrm{V}_I = [A \otimes W]^T \mathrm{V}_M \tag{1}$$

where V_I and V_M are 16 dimensional vectors corresponding to the 16 measurement intensities and 16 elements of the measured Mueller matrix of the sample, respectively. T denotes the transpose of the matrix, $\otimes$ denotes the Kronecker product. The instrument matrices W and A represent 4 Stokes vectors that characterize the different polarization states of the PSG and PSA, respectively. In particular, the polarization states of PSG and PSA are identical, which means $A = W$ [15,17,19]. The instrument matrix A is given by

$$\mathrm{A} = \frac{1}{2}\begin{bmatrix} 1 & A_{11} & A_{12} & A_{13} \\ 1 & A_{21} & A_{22} & A_{23} \\ 1 & A_{31} & A_{32} & A_{33} \\ 1 & A_{41} & A_{42} & A_{43} \end{bmatrix}^T \tag{2}$$

where the first and the second subscripts correspond to ith measurement intensity and jth Stokes parameter generated by PSA or PSG, respectively. The vector of Mueller matrix V_M can be estimated by [15]

$$\mathrm{V}_M = \left[A^{-1} \otimes A^{-1}\right]^T \mathrm{V}_I \tag{3}$$

The noise in each detected intensity is independent from the other statistically, the covariance matrix Γ_{V_I} of the vector V_I is a diagonal matrix, and each diagonal element equals to the variance of each detected intensity [15]. The variance of V_M can be characterized by its covariance matrix Γ_{V_M}

$$\Gamma_{\mathrm{V}_M} = \left[A^{-1} \otimes A^{-1}\right]^T \Gamma_{\mathrm{V}_I} \left[A^{-1} \otimes A^{-1}\right] \tag{4}$$

The ellipsometric parameters of isotropic reflecting surface and some anisotropic thin film samples can be calculated by only four elements that indicated in the following the block diagonal Muller matrix.

$$M = \begin{bmatrix} m_{11} & m_{12} & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & m_{33} & m_{34} \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix} \tag{5}$$

where the symbol "•" refers to the matrix elements irrelevant to the ellipsometric parameters, and σ_i^2 corresponds to the variance of the four elements (m_{11}, m_{12}, m_{33}, m_{34}).

The ellipsometric parameters Ψ and Δ of an isotropic reflecting surface or anisotropic optical material can be measured by MMP. Ψ and Δ are defined by the complex Fresnel coefficients r_p and r_s (r_p is the complex Fresnel coefficient parallel to the incident plane and r_s is that perpendicular to the incident plane). $\Psi = \tan^{-1}|r_p/r_s|$ is the amplitude ratio upon reflection, and $\Delta = \delta_p - \delta_s$ is the difference in phase shift. The ellipsometric parameters Ψ and Δ can be obtained from the partial Mueller matrix described in Eq. (5):

$$\Psi = \frac{1}{2}\cos^{-1}\left[\frac{-m_{12}}{m_{11}}\right],\ \Delta = \tan^{-1}\left[\frac{m_{34}}{m_{33}}\right] \tag{6}$$

To minimize the total variance of the four elements in Mueller matrix, the optimal instrument matrix should satisfy the following relation

$$A = arg\min \sum_{i \in \Omega} \sigma_i^2,\ \Omega = \left\{1, 2, 11, 12\right\} \tag{7}$$

If the dominant noise is Gaussian noise, the covariance matrix Γ_{V_I} is a diagonal matrix with the diagonal elements equal to the variance of Gaussian noise σ^2. The variance of each element of V_M only depend on the instrument matrix A and it is given by

$$\sigma_i^2 = \sigma^2\left[\left[AA^T\right]^{-1} \otimes \left[AA^T\right]^{-1}\right]_{ii},\ \forall i \in \Omega \tag{8}$$

If the dominant noise is Poisson shot noise, the variance of each element of V_M is related to the first element of V_M and it can be expressed by

$$\sigma_i^2 = \frac{[\mathrm{V}_M]_1}{4}\left[\left[AA^T\right]^{-1} \otimes \left[AA^T\right]^{-1}\right]_{ii},\ \forall i \in \Omega \tag{9}$$

By using Eq. (7) and Eq. (8), the ideal instrument matrix with the immunity to both Poisson and Gaussian noise can be obtained [30]

$$A^*_{4\text{-}elems} = \frac{1}{2}\begin{bmatrix} 1 & 0.443 & 0.732 & 0.518 \\ 1 & 0.443 & -0.732 & -0.518 \\ 1 & -0.443 & 0.732 & -0.518 \\ 1 & -0.443 & -0732 & 0.518 \end{bmatrix}^T \tag{10}$$

The last three elements in each row of the ideal instrument matrix are coordinates of eigenvectors of the MMP system. These normalized vectors consist a tetrahedron inside a Poincaré sphere of unit radius, which is shown in Fig. 1. It can be seen that the tetrahedron are regular, which means the configurations described by the instrument matrix of Eq. (10) are optimized and have the best noise immunity to the Poisson shot noise and Gaussian noise [31–33]. However, the existing practical configurations cannot generate the ideal measurement matrix so far.