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# Self-consistent one-dimensional electron system on liquid helium suspended over a nanoscale dielectric substrate

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ABSTRACT

For electrons above a superfluid helium film suspended on a specially designed dielectric substrate, $z = h(y)$, we obtain that both the transverse, along $z$, and the lateral, along $y$, quantizations are strongly enhanced due to a strong mutual coupling. The self-consistent quantum wires (QWs) with non-degenerated one-dimensional electron systems (1DESs) are obtained over a superfluid liquid helium (LH) suspended self-consistently on different dielectric substrates with a nanoscale modulation. A gap $\gtrsim$10 meV ($\gtrsim$1 meV) is obtained between the lowest two electron levels due to mainly the transverse (lateral) quantization. Our analytical model takes into account a strong interplay between the transverse and the lateral quantizations of an electron. It uses that the characteristic length (energy) along the former direction is essentially smaller (larger) than the one along the latter, in a close analogy with the adiabatic approximation.

## 1. Introduction

Since pioneering works [1–3] quantized states of electrons above LH suspended on different substrates are the subject of a strong ongoing interest [4–23]. Electrons floating on LH have been proposed for quantum computing in a seminal work Ref. [10]. For a plane substrate and a large thickness of LH film, $d \gtrsim 0.5\,\mu\text{m}$, any effect of the substrate is negligible [4,6]. This allows a two-dimensional electron system (2DES) on a bulk LH and a single electron on a bulk film [10] with a 1D hydrogenic spectrum [4–6,10] $E_m^{1H} = -R/m^2$. Here $R \approx 8\,\text{K}$ is an effective Rydberg energy. For quantum computing in Ref. [10] it is suggested to patten the bottom electrode with features spaced close to $d$ ($\approx 0.5\,\mu\text{m}$). So that each feature traps one electron. Metallic posts submerged by the depth $\sim 0.5\,\mu\text{m}$, beneath practically plane helium surface, are suggested [12]. They form quantum dots for electrons on LH which may serve as the qubits of a quantum computer [12], in particular, at temperature $T \approx 10\,\text{mK}$ [10,12]. Surface electrons with band-type spectrum on LH over metallic periodic substrate of the diffraction grating type are proposed by Ginzburg and Monarkha [7]. Where an amplitude of modulation is much smaller than $d$ and a free surface of LH is assumed as flat.

Electrons in a micron-scale and a nanoscale channels filled by capillary action with LH [5,8–10,13–23] attract recently much attention, in particular, due to their high potential in creating qubits with the needed properties of performance. The systems of such channels are promising for construction of the equivalent of a charge-coupled device (CCD) [24] that, in addition, will allow the large scale transport of qubits [15,16,19]. In interesting experiments of Refs. [16,19] electrons are studied in the channels of a width $\gtrsim 3\,\mu\text{m}$ at $T \approx 1.5\,\text{K}$. Theoretical framework of Refs. [16,19] treats electrons mainly as ones above a bulk LH.

Indeed, usually some important characteristics of electrons in these channels such as a gap between the lowest electron levels of the transverse (lateral) quantization, a form of the lateral potential, a form of the LH surface, a lateral density profile, etc. are not well known [5,8,9,14–17,19–23]. In particular, due to absence of interplay and self-consistency between transverse and lateral quantizations within used theoretical frameworks.

In present study self-consistent 1DESs in QWs over LH suspended on different nanoscale dielectric substrates are obtained, for $T = 0.6\,\text{K}$. A strong interplay between the quantizations of an electron along $z$ and $y$ directions is treated within present approach. It uses, in particular, some analogies with well known adiabatic approximation [25]. In Fig. 1 the sketch of a geometry of studied model is shown. We consider that a dielectric substrate is periodic along the $y$-direction with a finite period $\Delta L_y$ (unless otherwise stated) and $L_x \to \infty$. Within the main super cell ($L_x \times \Delta L_y$; $|y| \le \Delta L_y/2$) the substrate profile $z = h(y)$ is assumed as

$$h(y) = 0, \quad a/4 \le |y| \le \Delta L_y/2\ ,$$
$$h(y) = h_1 \cos(2\pi y/a), \quad |y| \le a/4\ , \tag{1}$$

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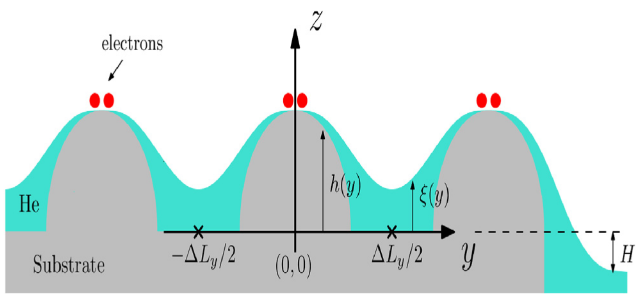


**Fig. 1.** A sketch, not to scale, of a model geometry. Substrate surface $z = h(y)$, LH surface $z = \xi(y)$, and LH thickness $d(y) = \xi(y) - h(y)$.

where $h_1$, $a$, $\Delta L_y$ are the characteristic scales of its modulation, cf. Fig. 1. We assume that $2h_1/a \ll 1$. I.e., a substrate profile is smooth.

For obtained systems of QWs, within the main super cell a 1DES is laterally localized at $y = 0$, cf. Fig. 1. Point out that effect of tunnel coupling between 1DESs of neighboring super cells is negligible for present systems of QWs. We have obtained a strong “long-range” effect of $\Delta L_y$ on the properties of a self-consistent 1DES at the region $10\,\mu m \ge \Delta L_y \ge 1\,\mu m$. For a given linear density within a super cell $n_L = N_{tot}/L_x$; $N_{tot}$ is the total number of electrons within a super cell. It is related with an essential dependence of LH profile within this region. That induces a strong modification of the transverse and the lateral quantizations for an electron. In particular, an essential modification of the effective electron potential is obtained due to a strong change for the image potential of substrate.

Notice, for $\Delta L_y \gtrsim 50\,\mu m$ properties of a self-consistent 1DES become practically independent of $\Delta L_y$. In present figures we assume that $\Delta L_y = 1\,\mu m$ or $10\,\mu m$. Then obtained results can be applied to the properties of QWs within a finite region $|y| \le L_y/2$, with $L_y \ge \Delta L_y$, if the substrate have a finite region of periodic modulation $|y| \ge L_y/2 + 25\,\mu m$.

In Subsection 2.1 we present a self-consistent Hamiltonian of an electron on a self-consistent LH film, suspended over a dielectric substrate with a nanoscale lateral modulation. In Subsection 2.2 we give the rest of a self-consistent framework for our model. It defines a self-consistent profile of LH suspended on the dielectric substrate, for a given linear density within a super cell. In Section 3 we present results and discussions on the self-consistent profiles of LH films suspended on the special dielectric substrates, the lowest levels of the transverse and of the lateral quantizations, a self-consistent electron density $n(y)$ profiles of 1DESs in obtained self-consistent electron nano-channels. Conclusions follow in Section 4.

## 2. Self-consistent model of electrons over liquid helium on a substrate with nanoscale modulation

### 2.1. One-electron Hamiltonian

We consider that between a surface of LH, $z = \xi(y)$, and the surface of substrate, $z = h(y)$, Eq. (1) a LH film is formed of the thickness $d(y) = \xi(y) - h(y) > 0$. First we assume that $\Delta L_y \to \infty$, later on we will show how our study can be extended to a finite $\Delta L_y$. Then the wave functions and the eigenvalues of an electron over LH are defined by the Schrodinger equation [5,6,14]

$$\left[-\frac{\hbar^2}{2m_0}\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}\right)+V(z,y)\right]\times\Psi_\beta(z,y,x)=W_\beta\Psi_\beta(z,y,x), \tag{2}$$

where three quantum numbers $\beta = \{k_{x\beta}, n_{y\beta}, n_{z\beta}\}$ are given by the wave number $k_{x\beta} = 2\pi n_{x\beta}/L_x$ and two integer quantum numbers $n_{y\beta} = 1, 2, 3, \ldots$, $n_{z\beta} = 1, 2, 3, \ldots$. As we assume the Born-von Karman boundary condition along $x$, we have $n_{x\beta} = 0, \pm 1, \pm 2, \ldots$. In Eq. (2), e.g., following Refs. [5,6,14], we have that

$$V(z,y) = -\frac{\Lambda}{z-\xi(y)} - \frac{\Lambda_1}{z-h(y)} + \left|e\right|E_p z, \tag{3}$$

where $\Lambda = e^2(\varepsilon_{LH}-1)/[4(\varepsilon_{LH}+1)]$ and $\Lambda_1 = e^2(\varepsilon_S-1)/[4(\varepsilon_S+1)]$. Here $\varepsilon_{LH} \approx 1.054$ is the dielectric constant of LH, $\varepsilon_S$ is the dielectric constant of substrate, and $E_p$ is an external (also called as holding) electric field. The first two terms in the right hand side of Eq. (3) represent the main contributions to the image potential energy [6]. The former term represents the image potential energy due to a bulk LH and the latter one shows a main effect of the substrate (for an infinite thickness of a LH film it is nullified).

Point out, Eq. (3) can be considered as exact if $\xi(y)$ and $h(y)$ are the linear polynomial functions of $y$ or independent of $y$. For more complex dependences of $\xi(y)$ and $h(y)$ on $y$, Eq. (3) is valid if $h(y)$ is smooth enough within an actual region. Where an electron is present mainly. This justifies the second term in Eq. (3). Further, the first term in Eq. (3) is readily justified due to a smoother $\xi(y)$ than $h(y)$ and closer average position of an electron along $z$ to the characteristic boundary. Here it is the LH surface $\xi(y)$. I.e., in Eq. (3) an electron image potential is well approximated by the first two terms of the right hand side provided the distance between the electron and the dielectric is small relative to the curvature of the dielectric surface.

As potential Eq. (3) is independent of $x$ we look for a solution of Eq. (2) as follows

$$\Psi_\beta(z,y,x) = L_x^{-1/2} e^{ik_{x\beta}x}\, \psi_{n_{z\beta},\,n_{y\beta}}(z,y)\,. \tag{4}$$

Then from Eq. (2) we obtain

$$\left[-\frac{\hbar^2}{2m_0}\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial y^2}\right)+V(z,y)\right]\psi_{n_{z\beta},\,n_{y\beta}}(z,y) = \widetilde{W}_{n_{z\beta},\,n_{y\beta}}\psi_{n_{z\beta},\,n_{y\beta}}(z,y), \tag{5}$$

where $\widetilde{W}_{n_{z\beta},\,n_{y\beta}} = W_\beta - \frac{\hbar^2 k_{x\beta}^2}{2m_0}$.

To solve Eq. (5) we develop an approach similar with the well known adiabatic method [25], that separates a fast movement of electrons from a slow movement of nuclei, to separate a fast movement along z-axis, on a short space scale $\Delta z$, from a slow movement along y-axis, on the scale $\Delta y \gg \Delta z$. We assume that

$$\psi_{n_{z\beta},\,n_{y\beta}}(z,y) = \Phi_{n_{y\beta}}(y)\varphi_{n_{z\beta}}(z,y), \tag{6}$$

where $\varphi_{n_{z\beta}}(z,y)$ is a real function (this condition always can be satisfied as it is a discrete spectrum state; $n_{z\beta} = 1, 2, \ldots$) that satisfies

$$\left[-\frac{\hbar^2}{2m_0}\frac{\partial^2}{\partial z^2}+V(z,y)\right]\varphi_{n_{z\beta}}(z,y) = \mathscr{E}_{n_{z\beta}}(y)\varphi_{n_{z\beta}}(z,y), \tag{7}$$

where $y$ has the role of a parameter. Then, substituting Eq. (6) in Eq. (5) and using Eq. (7), we obtain

$$-\frac{\hbar^2}{2m_0}\left[\varphi_{n_{z\beta}}(z,y)\frac{\partial^2}{\partial y^2}\Phi_{n_{y\beta}}(y) + 2\frac{\partial\Phi_{n_{y\beta}}(y)}{\partial y}\times\frac{\partial\varphi_{n_{z\beta}}(z,y)}{\partial y} + \Phi_{n_{y\beta}}(y)\frac{\partial^2}{\partial y^2}\varphi_{n_{z\beta}}(z,y)\right] = (\widetilde{W}_{n_{z\beta},\,n_{y\beta}} - \mathscr{E}_{n_{z\beta}}(y))\Phi_{n_{y\beta}}(y)\varphi_{n_{z\beta}}(z,y). \tag{8}$$

As a wave function of discrete spectrum $\varphi_{n_{z\beta}}(z,y) = 0$, for $z \le \xi(y)$, and it is localised at $z \approx \xi(y)$ (e.g., within a few nanometers from the LH surface for typical conditions of below Figs. 2–11), we obtain from its normalization

$$\int_{-\infty}^{\infty} dz\, \varphi_{n_{z\beta}}^2(z,y) = 1, \tag{9}$$

after applying $\partial/\partial y$, that

$$\int_{-\infty}^{\infty} dz\, \varphi_{n_{z\beta}}(z,y)\partial\varphi_{n_{z\beta}}(z,y)/\partial y = 0. \tag{10}$$

Then multiplying Eq. (8) by $\varphi_{n_{z\beta}}(z,y)$ and integrating over z, $\int_{-\infty}^{\infty} dz$,