Physica E: Low-dimensional Systems and Nanostructures 101 (2018) 22–28

Contents lists available at ScienceDirect

Physica E: Low-dimensional Systems and Nanostructures

journal homepage: www.elsevier.com/locate/physe

Self-similar transmission patterns induced by magnetic field effects in graphene

R. Rodríguez-González [a], I. Rodríguez-Vargas [a,*], D.S. Díaz-Guerrero [a], L.M. Gaggero-Sager [b]

[a] Unidad Académica de Física, Universidad Autónoma de Zacatecas, Calzada Solidaridad Esquina con Paseo la Bufa S/N, 98060, Zacatecas, Zac., Mexico
[b] Centro de Investigación en Ingeniería y Ciencias Aplicadas, Universidad Autónoma del Estado de Morelos, Av. Universidad 1001, Col. Chamilpa, 62209, Cuernavaca, Morelos, Mexico

ARTICLE INFO

Keywords:
Self-similarity
Scaling rules
Transmittance
Magnetic field
Cantor structures

ABSTRACT

In this work we study the propagation of Dirac electrons through Cantor-like structures in graphene. In concrete, we are considering structures with magnetic and electrostatic barriers arrange in Cantor-like fashion. The Dirac-like equation and the transfer matrix approach have been used to obtain the transmission properties. We found self-similar patterns in the transmission probability or transmittance once the magnetic field is incorporated. Moreover, these patterns can be connected with other ones at different scales through well-defined scaling rules. In particular, we have found two scaling rules that become a useful tool to describe the self-similarity of our system. The first expression is related to the generation and the second one to the length of the Cantor-like structure. As far as we know it is the first time that a special self-similar structure in conjunction with magnetic field effects give rise to self-similar transmission patterns. It is also important to remark that according to our knowledge it is fundamental to break some symmetry of graphene in order to obtain self-similar transmission properties. In fact, in our case the time-reversal symmetry is broken by the magnetic field effects.

1. Introduction

In nature many peculiar features of certain phenomena are observable only under special conditions. For instance, recently by breaking either the time-reversal symmetry or the inversion symmetry novel materials such as topological insulators [1–3], Dirac semimetals [4,5], Weyl semimetals [6,7] and materials with special charge carriers like Kane electrons [8,9] have arisen. Then, the set of symmetries in a material (chiral symmetries) and specially its breaking (chiral symmetry breaking) can give rise to unprecedented materials with exotic properties. In fact, in graphene it has been shown that chiral symmetry breaking can change the character of the material from a semimetal to a strong insulator [10]. Even a metallic or superconducting phase can be induced by breaking some particular chiral symmetry. Actually, if we take into account the variety of 2D materials available today as well as the symmetry-breaking possibilities the opportunities for exotic materials are superb.

On the other hand, the two-dimensional nature of graphene constitutes an unprecedented platform to study the transmission and transport properties in special (self-similar) geometries such as those that can be constructed using the Sierpinski carpet, Cantor set, Koch curve, etc. In principle, these peculiar geometries can be obtained by nanostructuring the material. In fact, it is possible to obtain potential profiles with self-similar characteristics. Even, the profiles can have scaling in both the spatial and energy axis. These self-similar potential profiles were originally proposed in the context of semiconductor quantum wells [11,12]. Actually, in graphene we have several mechanisms to nanostructuring. Among the most relevant ones we can find those based on metallic electrodes [13–16], interacting substrates [17–20], strain [21–23] and ferromagnetic gates [24–31]. All these mechanisms modify the fundamental properties of graphene. For instance, if we have graphene on an interacting substrate such as SiC or hBN the dispersion relation is modified and most importantly a bandgap is induced. The interaction of the graphene sheet with the substrate breaks the intrinsic sublattices symmetry in graphene and consequently a bandgap opening arises. In addition, as a result of the symmetry breaking the pseudo-spin is not longer conserved as well as Klein tunneling is prevented [32,33]. In the case of metallic and ferromagnetic gates the associated electric

* Corresponding author.
E-mail address: isaac@fisica.uaz.edu.mx (I. Rodríguez-Vargas).

https://doi.org/10.1016/j.physe.2018.03.007
Received 13 November 2017; Received in revised form 31 January 2018; Accepted 7 March 2018
Available online 7 March 2018

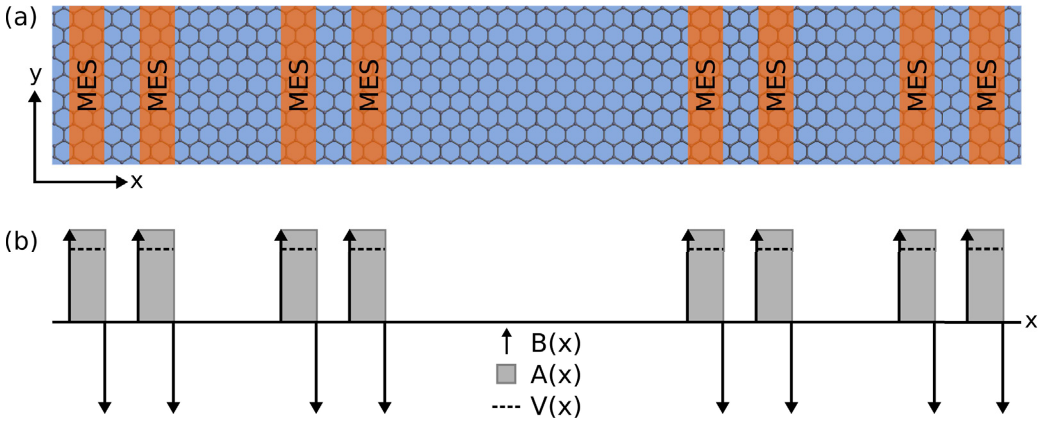

Fig. 1. (a) Schematic representation of the top view of a Cantor-like graphene-based structure under magnetoelectric effects. Graphene is placed on a non-interacting substrate like SiO_2 (shaded blue area). The magnetoelectric strips (MESs) are incorporated on top of graphene to tune the distribution and shape of the magnetic and electric fields applied perpendicularly to graphene and consequently the profile of magnetoelectric barriers. (b) Corresponding vector and scalar potential profiles for (a). The deltaic magnetic field is depicted as up and down arrows. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

and magnetic fields shift the graphene's Dirac cones in the energy and wavevector axis, respectively. In the case of the magnetic field, it also breaks a fundamental symmetry, specifically the time-reversal symmetry.

Under this context, the relativistic character of the charge carriers in graphene, the exotic properties that can arise due to the breaking of symmetries and the special geometries that can be imposed to graphene and other 2D materials by nanostructuration can confabulate to give place to unprecedented transmission and transport properties. In fact, in recent years, self-similar transmission and transport in graphene Cantor-like structures have been reported [34–37]. The self-similar transmittance and conductance patterns found obey well-defined scaling rules, that is, the patterns for different sizes of the system can be connected. The size of the system in the energy and spatial coordinates can be controlled by the generation and effective width of the system as well as the height of the barriers. Actually, the scaling rules correspond to precisely those parameters. Other important aspect to remark is that in order to obtain the mentioned self-similar patterns it is fundamental that the sublattices symmetry be broken, which correspond to structures with interacting substrates. Because as far as we have corroborated the self-similar characteristics are not present in structures in which the sublattices symmetry is preserved [35], i.e. structures in which the energy barriers are generated with metallic electrodes.

In this work, we study the transmission properties of graphene Cantor-like structures. In concrete, we explore the consequences of breaking the time-reversal symmetry. In order to induce the time-reversal symmetry breaking and at the same time obtain a self-similar (Cantor-like) structure we have considered that the energy barriers that composed the structure are generated by magnetic and electric fields. The magnetic field assures us the breaking of the time-reversal symmetry. The Dirac-like equation and the transfer matrix approach are implemented to describe the charge carriers and to obtain the transmission properties, respectively. We obtain that once the magnetic field is incorporated the transmission patterns show self-similar characteristics. Even more important, we obtain scaling rules that can describe the self-similar transmission patterns at different scales. To our knowledge this is the first time that scaling rules are reported under magnetic field effects.

2. Methodology

Our Cantor-like structure is composed of a graphene sheet placed on a non-interacting substrate like SiO_2. Magnetoelectric strips are considered as top gates in order to generate the magnetic and electrostatic (magnetoelectric) potential barriers along the structure, see Fig. 1. In fact, ferromagnetic strips were successfully deposited on semiconductors heterostructures [38] and constitute one of the main proposals to obtain magnetic barriers in graphene [28]. So, in principle, the strips allow us to induce different profiles for the magnetoelectric barriers. In our specific case we are considering step-wise scalar and vector potential barriers, Fig. 1 (b). These barriers are arranged according to the construction rules of the Cantor set in order to obtain our self-similar structure. A schematic representation (top view) of our Cantor-like structure is shown in Fig. 1 (a). The blue region and the orange stripes represent the SiO_2 substrate beneath the graphene sheet and the top magnetoelectric gates, respectively. Under these conditions we are dealing with two different regions corresponding to those without and with magnetoelectric barriers. The physics in these regions can be described by the corresponding Dirac-like equation. In fact, the Hamiltonian that corresponds to regions with magnetoelectric barriers is given by:

$$H = v_F \boldsymbol{\sigma} \cdot (\mathbf{p} + e\mathbf{A}) + V(x)\sigma_0, \tag{1}$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$ are the Pauli matrices, $\boldsymbol{p} = (p_x, p_y) = i\hbar\nabla$ is the momentum operator, v_F is the Fermi velocity of the Dirac electrons in graphene, $\mathbf{A} = (0, A_y, 0)$ is the vector potential given in the Landau gauge, $V(x)$ is the scalar potential and σ_0 is the 2×2 unitary matrix.

For this particular problem, we have introduced the dimensionless quantities, $l_B = \sqrt{\hbar/eB_0}$ and $E_0 = \hbar v_F / l_B$ that refer to the strength and length of the magnetic field as well as the unit of energy, respectively. Here, B_0 is a magnetic field of reference that help us to define the basic units of energy and length. In all our numerical calculations a typical realistic value of $B_0 = 0.1$ T is used, with $l_B = 811$ Å and $E_0 = 7.0$ meV [39]. Then, $\mathbf{A}(x) = A_y\hat{y} = B(B_0)l_B\hat{y}$ and $V(x) = U_0$ are defined as the vector and scalar potentials. The magnetic field B comes in terms of B_0. By solving the Dirac-like equation that corresponds to Eq. (1) it is possible to obtain the following dispersion relation:

$$E = U_0 \pm \sqrt{\hbar^2 v_F^2 q_x^2 + v_F^2(\hbar q_y + eA_y)^2}, \tag{2}$$

the ± signs correspond to electrons and holes, respectively. Moreover, the wavefunctions take the form:

$$\psi_\pm(x, y) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ v_\pm \end{pmatrix} e^{\pm i q_x x + i q_y y}, \tag{3}$$

with

$$v_\pm = \frac{\hbar v_F \left(\pm q_x + i\left(q_y + \frac{e}{\hbar} A_y \right) \right)}{E - U_0}. \tag{4}$$

In addition, the wave vector in the propagation direction comes as:

$$q_x = \frac{1}{\hbar v_F} \sqrt{(E - U_0)^2 - v_F^2(\hbar q_y + eA_y)^2}. \tag{5}$$

In contrast, for regions without magnetic field the Hamiltonian come as:

$$H = v_F(\boldsymbol{\sigma} \cdot \mathbf{p}), \tag{6}$$